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An Attempt of Quantitative Interpretation of Streaming Potentials

Poskus količinskega vrednotenja strujnih potencialov

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The paper deals with an approximative quantitative interpretation of the surface self-potential anomalies, related with the underground water movement, so called streaming potential anomalies. From the relation between electrical and mechanical quantities, the dynamical equation of liquid motion, and the Darcy's law, three equations are deduced for an estimation of apparent hydrogeological parameters: apparent velocity, apparent permeability, and apparent gradient of water table. Thus, from the streaming potential anomalies, maps of these quantities could be done. Finally the problems associated with the proposed interpretation method are briefly described.

Avtor obravnava približno količinsko vrednotenje površinskih anomalij lastnega potenciala, ki jih povzroča gibanje podzemeljske vode. Izhajajoč iz znane zveze med električnimi in mehanskimi količinami ter iz Navier-Stokesove enačbe za gibanje tekočin in Darcyjevega zakona je avtor izpeljal tri enačbe za oceno navideznih hidrogeoloških parametrov: navidezne hitrosti, navidezne prepustnosti in navideznega gradienta vodne gladine. Iz karte lastnega potenciala je možno s pomočjo omenjenih enačb izdelati karte parametrov, oziroma njihovih relativnih vrednosti. Za izračun absolutnih vrednosti teh parametrov pa je treba neodvisno oceniti navidezni elektrokinetični parameter. Na koncu so na kratko opisane pomanjkljivosti predložene metode vrednotenja.

Introduction

Natural potential anomalies called »streaming potential anomalies« might offer an efficient guidance for treating underground water flow and for locating underground water outlets and inlets. Theoretical aspects and practical results of this method have been discussed by many scientists and explorers. V. A. Bogoslovsky and A. A. Ogil'vi (1970) have further shown how to use streaming potential anomalies as a quantitative index of the rate of seepage from water reservoirs.

In this paper, likewise a quantitative method available for interpretation of streaming potential anomalies is proposed. The aim is, how to deduce some hydrogeological and hydrological parameters from surface geophysical measurements. It is, however, more or less only an attempt how to interpret such anomalies in a simplified manner. Here a step further is done concerning the idea given in the previous paper (J. Lapajne, 1976), where the method of calculation of average parameters was treated.

Basic Equations

The method is based on the well known phenomenom of electrokinetic polarization caused by the underground water flow.

Let us take as a basis the Helmholtz's equation of the electrokinetic polarisation for the laminar fluid flow through a cylindrical tube

$$\text{grad } V_{sp} = \frac{\varepsilon \, \varepsilon_0 \, \rho}{\eta} \, \zeta \text{ grad } p \tag{1}$$

where V_{sp} — streaming potential (electrofiltration potential, filtration potential), i. e. a potential along water flow path,
ζ — electrokinetic potential (called also zeta-potential), i. e. a potential difference through the electrical double layer,
ε — dielectric constant of the liquid,
ε_0 — permittivity of free space,
ϱ — resistivity of the liquid,
η — viscosity of the liquid,
p — pressure in the cylinder.

For ground water movement in natural geological conditions electrokinetic relationships are more complicated. Thus the streaming potentials in porous and/or fissured medium also depend (M. U. Ahmad, 1964; V. A. Bogoslovsky, and A. A. Ogil'vi, 1972) on the granulometric composition, on the permeability of the medium, on the free surface area of pores and/or fissures, on the thickness of firm and loosely bound water films, on the salt content in water and on other factors.

Using the Poiseuille's law, we obtain from (1) the following relation in which all quantities, not important for futher treating, are ignored (see for instance A. P. Krajew, 1965):

$$\mathbf{E}_{sp} = \frac{1}{c} \mathbf{v}, \tag{2}$$

where $\mathbf{E}_{sp}$ — streaming electric field intensity,
$\mathbf{v}$ — flow velocity,
c — electrokinetic parameter.

The above equation is valid, as a rule, only for streamline flow. It has been not examined if it could be taken as a rough approximation also for a turbulent flow.

For the stationary movement ($\partial \mathbf{v} / \partial t = 0$) of nonviscous incompressible liquid Navier-Stokes equation is

$$d \, (\mathbf{v} \, \nabla) \, \mathbf{v} = d \, \mathbf{g} - \text{grad } p, \tag{3}$$

where d — liquid density, $\mathbf{g}$ — gravitational acceleration.

Using Darcy's law

$$\mathbf{v} = -\frac{k}{d\,g}\,\mathrm{grad}\,(p + d\,g\,z) = \frac{k}{d\,g}\,(d\,\mathbf{g} - \mathrm{grad}\,p), \tag{4}$$

where k — coefficient of permeability, or simply »permeability«,
z — vertical coordinate of liquid level, i. e. depth of liquid level,

equation (3) becomes

$$k\,(\mathbf{v}\,\nabla)\mathbf{v} = g\,\mathbf{v}. \tag{5}$$

From (2) and (5), we have

$$k\,(\mathbf{E}_{sp}\,\nabla)\,(c\,\mathbf{E}_{sp}) = g\,\mathbf{E}_{sp}. \tag{6}$$

The equation (6) could be rewritten in the form:

$$k\,c\,(\mathbf{E}_{sp}\,\nabla)\,\mathbf{E}_{sp} = [g - k\,(\mathbf{E}_{sp}\,\mathrm{grad}\,c)]\,\mathbf{E}_{sp}. \tag{7}$$

It was already pointed out that in natural geological conditions the streaming potential depends on many hydrogeological properties which are far from being unchangeable over the entire exploration field. Thus, the quantity $\mathbf{v}_{sp}/\mathbf{E}_{sp} = c$ is not constant. But if the changes in $\mathbf{v}_{sp}/\mathbf{E}_{sp}$ are small compared with changes in $\mathbf{E}_{sp}$, the term $k\,(\mathbf{E}_{sp}\,\mathrm{grad}\,c)$ in (7) is practicaly very likely small compared with g. If we neglect it, we get

$$k\,c\,(\mathbf{E}_{sp}\,\nabla)\,\mathbf{E}_{sp} = g\,\mathbf{E}_{sp}. \tag{8}$$

With (2), (4) (taking $p = 0$), and (8), we get

$$g\,\mathrm{grad}\,z = -c^2\,(\mathbf{E}_{sp}\,\nabla)\,\mathbf{E}_{sp}. \tag{9}$$

If it is possible to measure streaming potential and to evaluate the coefficient c, it would be possible to deduce from (2), (8) and (9) important hydrological quantities $\mathbf{v}$, k, and grad z and finaly the flow rate.

In ground geolectrical survey surface potential distribution $V = V\,(x, y)$ is measured, and from this $\mathbf{E} = \mathbf{E}\,(x, y)$ is deduced. Now, let us suppose the equations (2), (8), and (9) to be, in a very rough approximation, used also for the electric field on the surface. Thus, for the measured quantities these three equations are:

$$\frac{\mathbf{v}_a}{c_a} = -\mathrm{grad}\,V = \mathbf{E}, \tag{10}$$

$$k_a\,c_a = \frac{g\,\mathbf{E}}{(\mathbf{E}\,\nabla)\,\mathbf{E}}, \tag{11}$$

$$\frac{(\mathrm{grad}\,z)_a}{c^2_a} = -\frac{(\mathbf{E}\,\nabla)\,\mathbf{E}}{g}, \tag{12}$$

where $V = V\,(x, y)$ — self-potential (natural potential) on the surface, we may call it "surface streaming potential" (instead of "surface streaming potential anomaly" the term "streaming potential anomaly" is used),
$\mathbf{E} = \mathbf{E}\,(x, y)$ — electric field intensity on the surface,
$\mathbf{v}_a = \mathbf{v}_a\,(x, y)$ — apparent velocity,

$c_a = c_a\,(x, y)$ — apparent electrokinetic parameter,
$k_a = k_a\,(x, y)$ — apparent permeability,
$(\text{grad } z)_a = (\text{grad } z)_a\,(x, y)$ — apparent gradient of water level.

The equations (10), (11), and (12) are basic equations of the proposed approximate interpretation method of streaming potential anomalies. Using these equations, maps of $\mathbf{v}/c_a$, $k_a c_a$ *and* $(\text{grad } z)_a/c_a^2$ can be done. These maps could be interpreted as images of relative apparent velocities, relative apparent permeabilities and relative apparent gradients, if changes in these quantities are essentialy grater than changes in c_a.

To evaluate the absolute values of the above three apparent quantities, c_a must be found experimentaly. Mise-a-la-masse method might offer a resonable geophysical field procedure for this purpose. Hydrogeological investigations might also render necessary data for rough estimation of c_a.

Conclusions

In the above proposed interpretation method of streaming potentials, some doubtful suppositions, approximations and simplifications are present. We shall not discusse the validitiy of the relation (2). It must just be emphasized that this equation determines the "minimum roughness" of the interpretation.

Taking the electrokinetic parameter and the apparent electrokinetic parameter as quasi constant values over the entire exploration field could be a risky approximation, but this problem might be put off by dividing the exploration field into smaller regions, where these parameters could be taken as constants.

First uncertain presumption is the adoption of the equation (2) to the surface potential, i. e. the validity of equation (10).

Simplification with apparent quantities is very practical, but these quantities are a questionable idea of real quantities.

The most critical point is very likely the presupposition that certain self-potential anomaly is mainly a streaming potential anomaly.

Nevertheless, with a careful treatment the proposed quantitative interpretation method might find a practical use, while self-potential survey is very simple, fast and extremely cheap.

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