DEPTH DETERMINATION BY MEANS OF GRAVIMETRICAL SOUNDING*

by *Franc Šumi*

With 5 figures

By the gravimetrical interpretation it must be kept in mind that a certain mass distribution causes a definite gravimetrical anomaly. Conversely, any gravimetrical anomaly can be caused by various mass distributions. For a quantitative gravimetrical interpretation it is therefore necessary to get an assumption on the probable shape and size of the body, in order to be able to determine the position and the depth from gravity anomalies.

There are several well known methods of gravimetrical interpretation in the geophysical literature, by means of Bouguer anomalies, residual values, second derivatives etc. Possibilities of a quantitative interpretation and a depth estimation are described as well. In my paper: Analyse der Interpretation der gravimetrischen Messungen bei Erdöluntersuchungen, the possibility of a gravimetrical sounding along a cross profile above a two-dimensional body is mentioned. The principles and advantages of this method as well as its practical application will be discussed here.

By comparing the approximate equation for the residual value R (1) and the second derivative g_0'' (2) of the gravity on a profile above a two-dimensional body

$$R = g_0 - \bar{g_r} \tag{1}$$

$$g_0'' = \frac{2\,(g_0 - \bar{g_r})}{r^2} \tag{2}$$

it can be seen, that these equations are very similar. The only difference is the coefficient $2/r^2$ and the fact that the interval r in the equation (1) must be large, whereas in the equation (2) it is small. g_0 is the gravity at the examined point, $\bar{g_r}$ is the average value at the same point, defined as the arithmetical mean of gravity at two points at a distance r to the right and to the left from the point g_0.

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It should be noted that the equation (2) differs little from Mr Baranov's excellent equation (3) for the second derivative, calculated for a two-dimensional case,

$$g_0'' = \frac{1}{r^2}\left(\frac{103}{50}\,g_0 - \frac{104}{50}\,g_{\bar{r}} + \frac{1}{50}\,g_{\overline{2r}}\right) \tag{3}$$

where besides the average gravity value $g_{\bar{r}}$ at a distance r, the average value $g_{\overline{2r}}$ at a distance 2r is also considered.

When applying equation (2) for the second derivative, a corresponding interval r must be chosen. By two-dimensional bodies comparisons are made between theoretical values of the second derivative, computed directly from the known position of the body, and the second derivative computed by the use of the equation (2) for various intervals r. In the case of a horizontal cylinder, the interval r in the equation (2) must be less than ⅓ of the depth to the centre of the cylinder, in order to keep the error in the range of less than 10 % of the actual second derivative value. This is particularly important when second derivatives are used for the quantitative interpretation.

Similarly to the equations (1) and (2) for the residual value and the second derivative, we can develop an analogous equation for the vertical gravity gradient g_0' at the same point

$$-g_0' = \frac{g_0 - g_{\bar{r}}}{r} \tag{4}$$

The equation (4) is valid only for a certain interval r and depends on the position, the shape and the depth of the two-dimensional body. The interval r for a point directly above the centre of a cylinder should be equal to its depth. Other values of r correspond to other two-dimensional bodies.

The similarity between the equations (1), (2) and (4) was already mentioned. When one of these equations for various values of r is used, the functions (1), (2) and (4) are proportional to the second derivative, if r is small compared to the depth, to the first derivative by a certain r and to the residual value by a large r. When successive increased values of r are used in equation (4), the function g_0' reaches its maximum by r_{max} and depends on the shape and the depth of the body. Conversely, by a geologically supposed shape and an already obtained r_{max} it is possible to determine the depth of the centre of the body. Due to the analogy with the electrical resistivity method, where by way of the expansion of electrodes the depth penetration is increased, the present gravimetrical method is called gravimetrical sounding.

In order to simplify the calculation and the interpretation, the equation (4) may be written as follows

$$-g_0' = \left(\frac{g_0}{2r} - \frac{g_{\bar{L}}}{2r}\right) - \left(\frac{g_{\bar{R}}}{2r} - \frac{g_0}{2r}\right) = \frac{G_{\bar{L}} - G_{\bar{R}}}{2} \tag{5}$$

where g_L and g_R represent the gravity values to the left and to the right of g_0 at a distance r. $G_{\overline{L}}$ and $G_{\overline{R}}$ are the average horizontal gradients on a profile to the left and to the right from g_0, between points g_0 and g_L and between g_R and g_0. In order to be able to apply the already known equations for horizontal gradients of bodies of various shapes, the sounding is not done by means of the average gradients $G_{\overline{L}}$ and $G_{\overline{R}}$ as in equations (4) and (5), but by the use of the actual gradients G_L and G_R at a distance ϱ to the left and to the right from the point g_0. These gradients can be obtained either by close gravimeter measurements on the profile or by the torsion balance. In most cases sufficient accuracy in sounding is obtained by modern gravimeters. The torsion balance should be applied only for bodies near the surface.

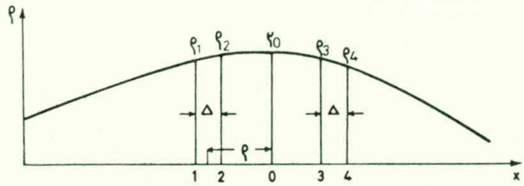

Fig. 1. A point arrangement for a gravimetrical sounding
1. sl. Razpored točk za gravimetrično sondiranje

The final equations for the gravimetrical sounding is

$$f(\varrho) = \frac{(g_2 - g_1) - (g_4 - g_3)}{2\triangle} = \frac{g_2 + g_3 - g_1 - g_4}{2\triangle} \qquad (6)$$

when gravimeter data are used, and

$$f(\varrho) = \frac{G_L - G_R}{2} \qquad (7)$$

for a torsion balance.

The arrangement of the points g_1 to g_4 can be seen on Fig. 1. G_L and G_R are the components of the horizontal gradients in the direction of the profile at the points, which are at a distance ϱ to the left and to the right from the point g_0.

Equations (6) and (7) show, that the possible regional gradient is automatically eliminated. This is particularly important in the case of close measurements with the torsion balance on the hilly country, where terrain corrections need to be determined only in the vicinity of stations. Cartographic corrections are almost equal for all stations and can be considered as a regional gradient.

In the electrical method bodies of good or poor conductivity are located by resistivity mapping. Their depth is then determined by way of resistivity sounding. Similarly we map gravimetrically after equation

(6) with small and constant $\triangle$ on the cross profile above a two-dimensional body. The direction of the cross profile is perpendicular to the geologically supposed or gravimetrically determined strike. When mapping by the use of the equation (6) a regional gradient is eliminated, so that the maxima lie directly above the bodies and are not displaced as on an original gravimetrical profile. The maxima are gravimetrically sounded by the use of the equation (6) or (7). Similarly to the electrical resistivity method, a gravimetrical sounding curve is designed. The vertical axis shows the expanding values ϱ, and the horizontal one the functions f (ϱ) (6).

On Fig. 2. the theoretical case of a horizontal cylinder is shown. Besides a gravity anomaly g, caused by a cylindrical mass, a strong regional effect is superposed and the maximum of g is displaced because

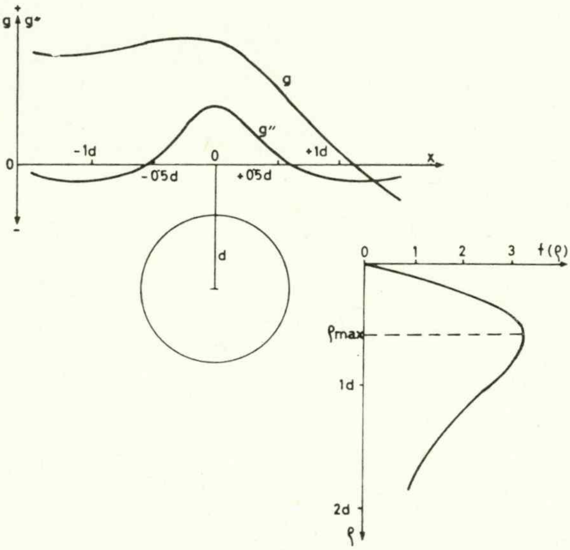

Fig. 2. Gravimetrical mapping and sounding above a horizontal cylinder
2. sl. Gravimetrično kartiranje in sondiranje nad horizontalnim valjem

of them. By applying a small value ϱ, the gravimetrical mapping gives a curve g″ (the approximate second derivative) with a maximum just above the cylinder. The sounding curve at this point is shown on Fig. 2. The function f (ϱ) has its maximum by ϱ_{max} and depends on the shape and the depth of the body.

For bodies of simple geometrical shapes the values ϱ_{max} which depend on the depth, can be calculated.

At the point above a sphere, the function f (ϱ) has a maximum for a value

$$\varrho_{max} = \frac{d}{2} \tag{8}$$

ϱ_{max} is determined after the same equation as x_{max} for the horizontal gradient above a sphere. ϱ_{max} obviously does not depend on the radius and the density of the sphere.

ϱ_{max} for a horizontal cylinder can be obtained in the same way (Fig. 2.)

$$\varrho_{max} = \frac{d}{\sqrt{3}} \tag{9}$$

In the case of a cylinder ϱ_{max} does not depend on the radius and the density.

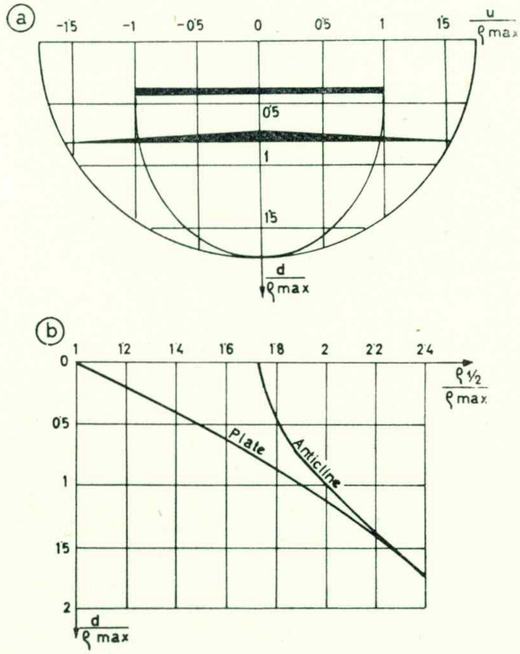

Fig. 3. Diagrams for gravimetrical sounding above a thin plate and a gentle anticline

3. sl. Diagram za interpretacijo tanke plošče in blage antiklinale z metodo gravimetričnega sondiranja

The relation (9) can be used for the gravimetrical sounding above a steep anticline. Equations for a thin plate and a flat triangle are derived for anticlines of gentle slopeness.

The position of the centre of a thin plate can be determined by gravimetrical mapping. A sounding curve at this point gives a value ϱ_{max} corresponding to $f(\varrho)_{max}$, and $\varrho_{1/2}$ where the function $f(\varrho)$ has a value of one half of $f(\varrho)_{max}$. In the sounding curve, two $f(\varrho)_{1/2}$ and the corresponding $\varrho_{1/2}$ are obtained. In all the equations only $\varrho_{1/2}$ which are greater than ϱ_{max} are considered. The depth d to the centre and the width 2u of the plate can be calculated from ϱ_{max} and $\varrho_{1/2}$.

$$\varrho^2_{max} = \frac{u^2 - d^2}{3} + \frac{2}{3}\sqrt{u^4 + u^2 d^2 + d^4} \qquad (10)$$

The diagram a on Fig. 3. is the graphical representation of the equation (10). From the diagram b the depth d can be taken. In the same diagrams the correnponding curve for a flat triangle are drawn. In the case of a flat triangle ϱ_{max} is evidently

$$\varrho_{max} = \sqrt{\frac{d^2 + u^2}{3}} \qquad (11)$$

The diagram a (Fig. 3.) shows, that all the triangles of the same value of ϱ_{max} are within the circle of a radius $\varrho_{max}\sqrt{3}$.

Both diagrams in Fig. 3. are strictly valid only, when the plate is infinitely thin and the triangle infinitely flat. When sounding after equations (6) and (7) above a thick plate or triangle, an error in the depth determination occurs, and depends upon (I) the relations between the width (2u) and the thickness (h) of the plate or the triangle, and upon (II) the relation between the depth to the centre of the plate or the triangle and the thickness. In the case of a plate the above mentioned relations may vary in wide limits, but the error remains negligible; it is not so in the case of a triangle.

In the case of a vertical dike of infinite depth extent, the point above the centre can be located by way of gravimetrical mapping. From the sounding curve at this point the depth d and the width 2u of the upper edge (Fig. 4.) can be determined by means of ϱ_{max} when the function f (ϱ) reaches its maximum, and $\varrho_{1/2}$ when the value of the function f (ϱ) is one half of the maximal. $\varrho_{1/2}$ greater than ϱ_{max} is considered only.

Relations between ϱ_{max}, $\varrho_{1/2}$, d and 2u are as follows

$$\varrho_{max} = \sqrt{u^2 + d^2} \qquad (12)$$

$$d = \frac{(\varrho_{1/2} - \varrho_{max})^2}{2\varrho_{1/2}} \qquad (13)$$

From the equation (12) it can be seen that the upper edge of the dike lies within the semicircle with the centre above the dike and the radius ϱ_{max}.

In the case of an inclined dike (Fig. 4.) the method of gravimetrical sounding has the advantage that the sounding curve at the point above the centre of the upper edge of the dike shows identical ϱ_{max} and $\varrho_{1/2}$ for vertical and for inclined dikes. Equations (12) and (13) are therefore extended to inclined dikes.

In order to determine the dip angle i of a dike, a series of sounding curves on the profile above a dike must be calculated, and a point with an absolute maximal value of the function f (ϱ) and the corresponding ϱ_{Max} must be found. The point with the absolute maximum f $(\varrho)_{\mathrm{Max}}$ lies on the side from the point O, on which a dike is inclined.

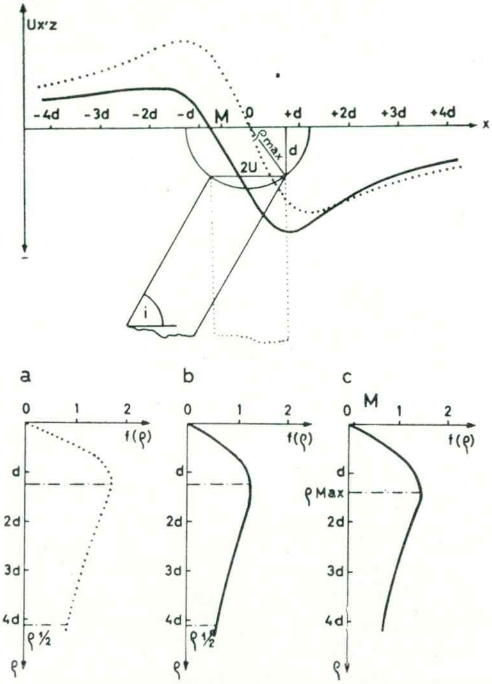

Fig. 4. Gravimetrical sounding above a vertical and an inclined dike of infinite depth extent

4. sl. Gravimetrično sondiranje nad vertikalnim in nagnjenim dajkom, ki vpada zelo globoko

The dip angle i is

$$\operatorname{ctg} i = \frac{\sqrt{\varrho^2_{\mathrm{Max}} - \varrho^2_{\mathrm{max}}}}{\dfrac{(\varrho_{1/2} - \varrho_{\mathrm{max}})^2}{2\varrho_{1/2}}} \tag{14}$$

In Fig. 4. a theoretical example of gravimetrical sounding above a vertical and an inclined dike adopted for the torsion balance is shown. The sounding curves a and b are calculated at the point O, just above the centre of the upper edge of the dike. The dotted curve a is obtained in the case of a vertical dike, the solid curve b in the case of an inclined dike. The values ϱ_{max} and $\varrho_{1/2}$ are equal for both curves. The depth a and the width 2u can be detemined by the use of equations (12) and (13). In order to find the dip angle i, sounding curves for a series of points on the profile above a dike are calculated and a gravimetrical sounding curve c at the point M with a maximal value $f(\varrho)_{Max}$ and corresponding ϱ_{Max} is found. The dip angle i is determined by the use of the equation (14). Using equations (6) and (7) in gravimetrical sounding, the regional effect is eliminated.

When two bodies of approximately equal size and depth are relativly close together, gravimetrical maxima above them are displaced and the apparent depth d' obtained by gravimetrical sounding differs from the actual depth d. The case of two parallel horizontal cylinders

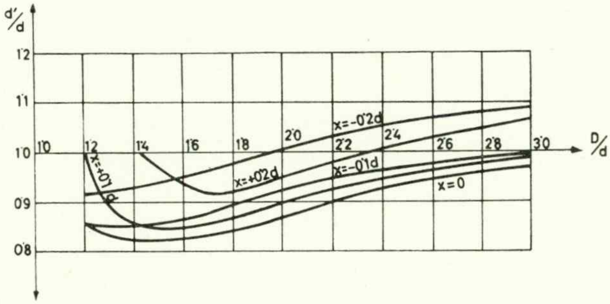

Fig. 5. Corrections for gravimetrical sounding in the case of two parallel cylinders

5. sl. Korekcije pri gravimetričnem sondiranju v primeru dveh vzporednih valjev

at different distances is analysed. By gravimetrical mapping on the profile, perpendicular to the strike, the position of both cylinders can be determined, when they do not lie to close. The distance D between the centres of the cylinders must be greater than the depth d. When sounding directly above a cylinder by the use of the equations (6) or (7), the regional effect of gravity is eliminated, but the gravimetrical effect of the other cylinder remains however and the obtained apparent depth d' differs from the actual depth d. The diagram on Fig. 5. shows the relations between the quotients d'/d and D/d in the case, (I) when the sounding point lies directly above the cylinder (x = 0), (II) when the sounding points is displaced for 10 and 20 % from the depth d in the direction of the other cylinder (x = + 0·1 d, x = + 0·2 d), and finally (III) when the sounding point is displaced for the same distance in the opposite direction (x = — 0·1 d, x = — 0·2 d). It can be seen, that the error for distances D, greater than 3 d is negligible. For distances D smaller than 3 d, the correction can be taken from the diagram on Fig. 5.

On Ulcinjsko Polje (Ulcinj Plain) in Montenegro, a bedrock of Cretaceous limestone is overlain by flysh and Tertiary beds. The density difference between limestone-flysh is 0·3 and between limestone-Tertiary

0·6. Gravimetrical measurements were carried out to locate the probable structures. The gravimetrical map on Fig. 6. shows a strong regional gradient in the north-west direction. The anticlines are therefore masked and can be recognized only by way of windings of the isogams. The general strike can be taken from the gravimetrical map. It is in good agreement with the geologically supposed NW—SE strike, which is cha-

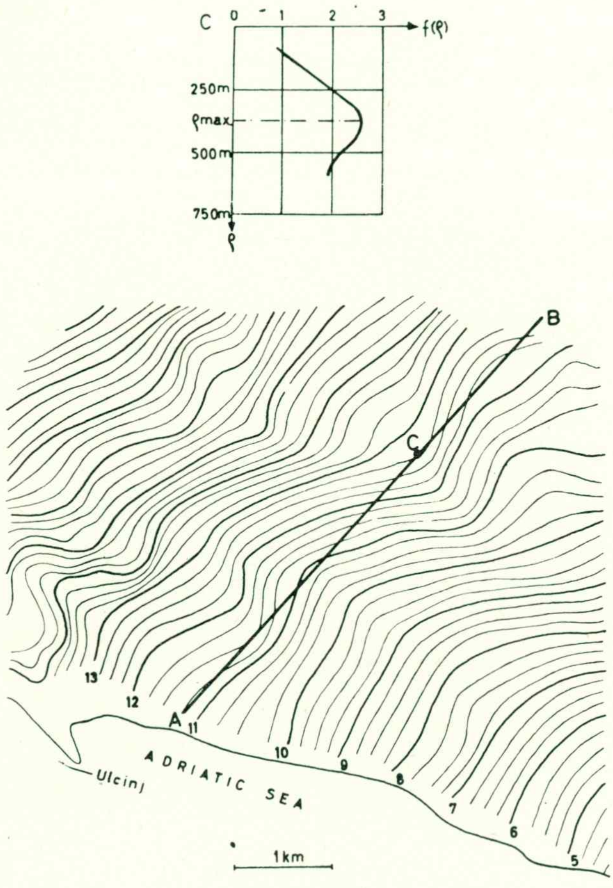

Fig. 6. Gravimetrical map of Ulcinjsko Polje. Sounding curve
6. sl. Gravimetrične meritve na Ulcinjskem Polju. Krivulja sondiranja

racteristic for the eastern Adriatic coast. The gravimetrical sounding curve at the point C on the profile A—B reaches its maximum by $\varrho_{max} = 370$ m (Fig. 6.). Geologically, steep anticlines are assumed, so that the equation (9) for a horizontal cylinder may be used for the determination of the depth. The depth d = 640 m to the centre of the anticline is obtained. Considering the geologically assumed shape of the anticline and the density difference, the detph to the top of the anticline can be estimated. Three parallel structures occur in the same field. The distance D from the point C to the southwestern structure is about 2200 m. Since the quotient D/d is greater than 3, the error in depth determination caused by the gravimetrical effect of a parallel structure can be neglected. Results obtained at bore holes agree with the gravimetrical data.

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DOLOČEVANJE GLOBINE S POMOČJO GRAVIMETRIČNEGA SONDIRANJA*

Povzetek

Pri interpretaciji gravimetričnih anomalij moramo upoštevati dejstvo, da iz znanega razporeda mas pod zemljo lahko izračunamo anomalno gravimetrično polje, obratno pa določenemu anomalnemu polju ustreza neskončno mnogo kombinacij razporedov mas, tako da za interpretacijo moramo privzeti neke predpostavke. Pri interpretaciji gravimetričnih podatkov uporabljamo višje odvode težnostnega potenciala, moremo pa, podobno kot pri geoelektričnem sondiranju, določiti globino telesa, ki povzroča težnostno anomalijo. Krivuljo gravimetričnega sondiranja (6) nanesemo v diagram kot funkcijo razdalje ϱ (1. sl.). Ko krivuljo interpretiramo, moramo vsaj približno vedeti, kakšno obliko ima telo, ki povzroča anomalijo. Često zadostuje, če vemo, ali so dimenzije telesa v vse smeri istega reda velikosti, ali pa je telo razpotegnjeno v neki smeri. Pri raziskavah na nafto so antiklinalne in sinklinalne strukture običajno razpotegnjene v neki smeri; pri raziskavah rudišč imajo rudna telesa popolnoma nepravilno obliko, ali pa obliko plošče. Da bi mogli interpretirati vse možne oblike teles, moramo obdelati limitne primere oblik: od krogle (8), preko horizontalnega valja (9), (2. sl.), do horizontalne plošče (10) in blage antiklinale (11), (3. sl.); razen horizontalne plošče pa še vertikalno (12 in 13) in nagnjeno (14), (4. sl.). Če leži več teles v bližini, moramo upoštevati tudi to (5. sl.).

Uporabo metode gravimetričnega sondiranja smo prikazali na primeru raziskav na Ulcinjskem Polju (6. sl.), kjer smo določili položaj in globino strmih antiklinalnih struktur.

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