



Quantified joint surface description and joint shear strength of small rock samples

Geometrijske lastnosti površine razpok in strižna trdnost po razpoki za manjše vzorce kamnin

Karmen FIFER BIZJAK & Andraž GERŠAK

Slovenian national building and civil engineering institute, Dimičeva ul. 12, SI-1000 Ljubljana, Slovenia;
e-mails: karmen.fifer@zag.si, andraz.gersak@zag.si

Prejeto / Received 31. 1. 2018; Sprejeto / Accepted 23. 4. 2018; Objavljeno na spletu / Published online 20. 7. 2018

Key words: camera-type 3D scanner, rock mechanics rock joint, roughness of the joints, rock joint shear strength

Ključne besede: 3D skener s kamero, mehanika hribin, hrapavost razpok, strižna trdnost kamnine po razpoki

Abstract

Geotechnical structures in rock masses such as tunnels, underground caverns, dam foundations and rock slopes often have problems with a jointed rock mass. The shear behaviour of a jointed rock mass depends on the mechanical behaviour of the discontinuities in that particular rock mass. If we want to understand the mechanical behaviour of a jointed rock mass, it is necessary to study the deformation and strength of a single joint. One of the primary objectives of this work is to improve the understanding of the frictional behaviour of rough rock joints under shear loads with regard to the roughness of the joint surface. The main problem is how to measure and quantify the roughness of the surface joint and connect the morphological parameters into a shear strength criterion. Until now, several criteria have been developed; however, all of them used large rock samples (20×10×10 cm). It is often not possible to get large samples, especially when the rock is under a few meters thick layer of soil. In this case, samples of rock can only be acquired with investigation borehole drilling, which means that the samples of rock are small and of different shapes. The paper presents the modified criterion that is suitable for calculating the peak shear stress of small samples.

Izvleček

Geotehnični objekti v hribini, kot so predori, podzemni prostori, pregrade in strme brežine, pogosto povzročajo težave zaradi različne razpokanosti hribinskega masiva. Strižno obnašanje celotnega hribinskega masiva je odvisno od razpok in njihovih strižnih lastnosti. Če želimo razumeti mehansko obnašanje hribinskega masiva, je potrebno preiskati strižne trdnostne karakteristike vsakega sistema razpok. Namen predstavljene raziskave je, da se preuči obnašanje razpok pod strižnimi obremenitvami v odvisnosti od hrapavosti površine razpok. Največji problem se pokaže pri meritvah hrapavosti razpok in povezave morfoloških parametrov površine razpoke s strižnimi karakteristikami same razpoke. Do sedaj je bilo predstavljenih že več kriterijev, vendar so bili vsi razviti na osnovi testiranja velikih vzorcev kamnine (20×10×10 cm). V večini primerov pa je velikih vzorcev kamnine nemogoče dobiti, predvsem takrat, ko je kamnina globoko pod več metrov debelo plastjo zemljine. V tem primeru se vzorce hribine lahko pridobi samo z raziskovalnim vrtanjem. Tako dobljeni vzorci pa so malih dimenzij in različnih oblik. V članku je predstavljen modificiran kriterij, ki je uporaben za izračun vrhunske strižne trdnosti v primeru, da imamo za raziskave dostopne samo vzorce manjših dimenzij.

Introduction

Geotechnical structures in rock material such as tunnels, underground caverns, dam foundations and rock slopes often have problems with a jointed rock mass. The shear behaviour of a jointed rock masses depends on the mechanical behaviour of the discontinuities in that particular

rock mass. If we want to understand the mechanical behaviour of a jointed rock mass, it is necessary to study the deformation and strength of a single joint. Until now, many experimental and numerical investigations have been carried out on the mechanical behaviour of rock joints (Barton, 1973; 1976; Barton & Choubey, 1977; Gras-

sellì & Egger, 2003; Hoek & Brown, 1980; Hoek & Bray, 1981; Hoek, 2000; Huang et al., 1992; Patton, 1966; Pellet et al., 2013).

The joint surface is one of the parameters that have the highest influence on the shear strength of the rock joint. Many parameters have been proposed over the past 40 years to describe the joint surface. Barton (1973) proposed a joint roughness coefficient (JRC) to quantify the roughness of a rock joint. The roughness profile of the nature rock joint is visually compared with 10 standard profiles suggested by Barton and Choubey (1977). However, the visual comparison method can be subjective without sufficient experience. The JRC has been widely used in rock engineering and is suggested by the International Society for Rock Mechanics (ISRM) as a useful parameter for describing the joint surface. In the last decade, several researchers published that the roughness of the rock joint could be somewhat underestimated (Hong et al., 2008; Lee et al., 2001).

Other methods using the fractal analysis (Kulatilake, et al., 2006; Odling, 1994) or the statistical approach (Reeves, 1985) have been used for identifying the rock joints. For all those methods, the two-dimensional (2D) description of the surface is used, although the joints have a three-dimensional morphology. Nowadays, with the advanced techniques, it is possible to measure and characterise the joint surface in three dimensions. The roughness metric based on the three-dimensional morphology was proposed by Grasselli (2001, 2002). The ATOS scanner was used for the accurate measurement of the joint roughness (Grasselli & Egger, 2003). The procedure of the roughness measurements is quite clear, yet the relationship between the joint mechanical properties and the geometric parameters is still the object of research nowadays. An empirical relation with the shear strength of the rock joints was studied and three-dimensional roughness parameters such as the contact area, the roughness parameter C and maximum dip angle θ_{max}^* were proposed for the calculation (Grasselli & Egger, 2003; Grasselli, 2006). All the parameters were determined by morphology functions. However, the anisotropy of rock joint was not considered in this criterion. Further research developed the modified peak shear strength criteria which could reflect the effect of dilatancy (Tang et al., 2014, 2015). The relationship between peak dilatancy angle and three-dimensional morphology characteristics was taken into consideration in these criteria. In the criterion proposed by Xia (2013), the variation

law of the dilatancy angle under various normal stresses was not inconsistent with the actual situation. Samples on which the shear tests were performed in all mentioned papers were of large dimensions, at least 200 cm². It is often not possible to get large samples, especially when the rock is under a few meters thick layer of soil. In this case, samples of rock can only be acquired with investigation borehole drilling, which means that samples of rock are small and of different shapes.

In the presented paper, the smaller samples from bore hole drilling were used for direct shear testing. For testing, the Robertson shear apparatus was used. That apparatus is limited by the size of the samples and by the height of normal and shear loads. Based on the experiment, a modified peak shear strength criterion was proposed, and a comprehensive criterion was developed for samples with smaller size and lower loads.

Methods

Use of a 3D Scanner

For measuring rock joint roughness, a camera-type digital three-dimensional scanner was used (fig. 1), which is a combined system with photogrammetry and fringe projection. It uses two cameras to capture the same position or asperity and can thus produce three-dimensional images showing the height of the asperity. Photogrammetry can be used for the measurement of sensor coordinates as well as for the global matching of partial views. In fringe projection, the projector illuminates the stripe of the patterned light on an object and two cameras capture the deformed shape of fringe by the object. An accurate roughness profile may be obtained by specific fringe characteristics. Therefore, the roughness underestimation of unevenness can be improved. Although this method requires a merging process because of image overlapping with “multi-viewing”, it produces a high resolution image quickly and conveniently (Reich et al. 2000; Lee & Ahn 2004).

While this method can quickly provide the high density cloud point, it is very sensitive to environmental conditions (Fifer, 2010).

The selected system for this study was Advanced Topometric Sensor (ATOS I) which combines photogrammetry and fringe projection. Because this system can yield high density three-dimensional point clouds for each image, it also requires a high computing system. ATOS has been used in the field of engineering

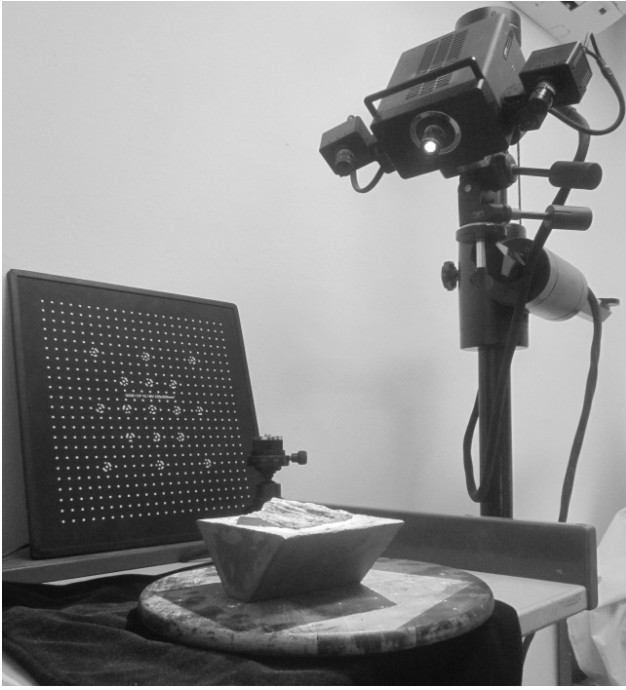


Fig. 1. The ATOS I 3D scanner and the sample.

for product digitization in industries such as the automotive industry. Details of the selected system are summarized in Table 1. The quality of surface measurements is very important to the estimation of roughness. The accuracy of the morphological model is dependent on the density of measurement points, measurement resolution and the precision with which these points can be located in space.

Table 1. The properties of the optical scanning system ATOS I.

Item	Value
Measured Points	800.000
Measurement Time (seconds)	0.8
Measuring Area (mm ²)	125 × 100 - 1000 × 800
Point Spacing (mm)	0.13-1.00
Measuring volume (mm ³)	125×100×90 to 1000×800×800
Measuring points per individual scan	1032×776 pixels

The camera-type 3D scanner has several advantages:

- the scanning process is fast and the image is accurate,
- the large scale of the specimen can be digitized,
- the scanning process can be performed in the field,
- the rock surface is not damaged during digitizing.

Calculation methods

The morphological parameters which we acquired with the scanning of samples were used for further calculation. The peak shear strength of samples was calculated according to several criteria which have been developed until now.

Grasselli (2001) proposed the apparent dip angle to calculate the three-dimensional morphology parameters (fig. 2). The average inclination angle is used according to the results of his research

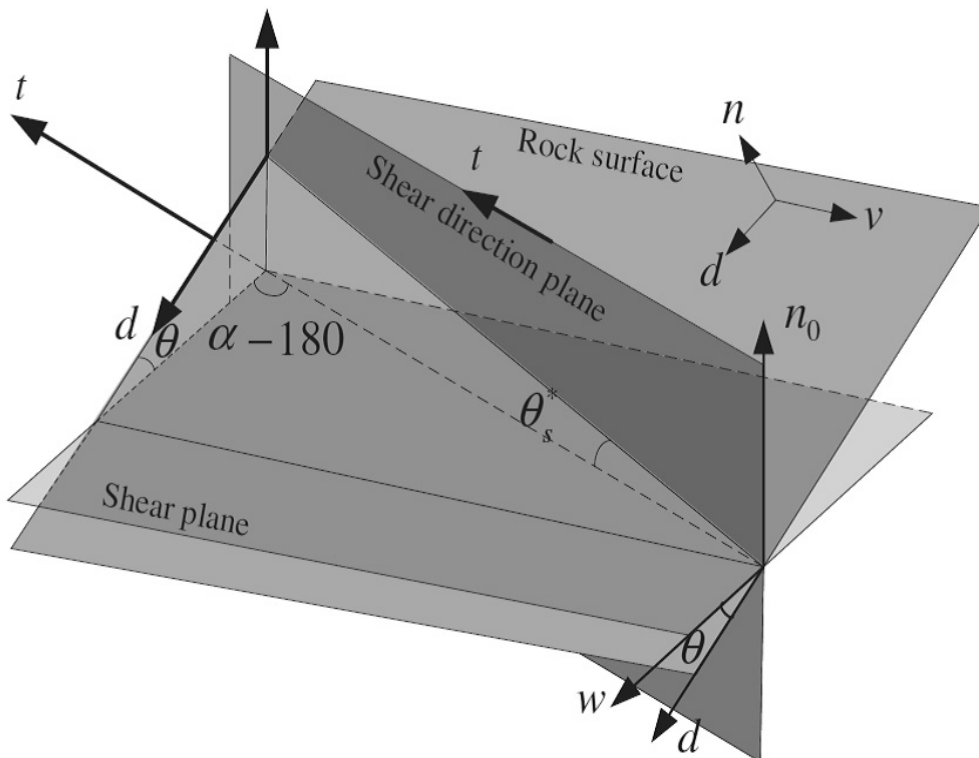


Fig. 2. Calculation of the 3D average angle of the rock joint surface (Grasselli, 2001). Grasselli (2001) proposed the criterion (G01) for calculating the peak shear stress according to the eq. 6.

$$\tan \theta_s^* = \tan \theta (-\cos \alpha) \quad (1)$$

$$\cos \theta = \frac{nn_o}{|n||n_o|} \quad (2)$$

$$\cos \alpha = \frac{tn_1}{|t||n_1|} \quad (3)$$

$$\bar{\theta}^* = \frac{tn_1}{|t||n_1|} \frac{1}{m} \sum_{i=1}^m \theta_{si}^* \quad (4)$$

where m is the number of triangles, θ_{si}^* is the apparent dip angle of the surface unit, α is the azimuth, t is the tilt angle, t is the shear direction vector, n is the outward normal vector of the triangle, n_o is the outward normal vector of the plane (see Fig. 2) and n_l is the projection vector of n . The maximum contact area is calculated as follows:

$$A_0 = \frac{A_l}{A_m} \quad (5)$$

where A_l is the sum of the area facing to the shear direction (θ_{si}^* is greater than zero) and A_m is the actual area of the whole joint surface.

$$\tau_p = \sigma_n * \tan(\varphi_r^*) * \left(1 + \exp\left(-\frac{1}{9A_0} * \frac{\theta_{max}^*}{C} * \frac{\sigma_n}{\sigma_t}\right) \right) \quad (6)$$

where s_n is the normal stress, s_t is tension strength of the rock and f_r a residual angle of friction, θ_{max}^* is the maximum apparent dip angle of the surface with respect to the shear direction, C is the roughness parameter, calculated using a best-fit regression function, which characterises the distribution of the apparent dip angles over the surface.

The next version of the same criterion (G06) includes parameter β which is the angle between the plane of schistosity and normal to the sample surface and where f_b is the basic angle of friction got from the direct shear test in laboratory.

$$\tau_p = \sigma_n \left[1 + \exp\left(-\frac{1}{9A_0} * \frac{\theta_{max}^*}{C} * \frac{\sigma_n}{\sigma_t}\right) * \tan\left[\varphi_b + \left(\frac{\theta_{max}^*}{C}\right)^{1,18\cos\beta}\right] \right] \quad (7)$$

A peak shear strength criterion (X13) with the use of a form of the Mohr-Coulomb equation was developed by Xia (Xia, 2013). The criterion is presented with eq. 8.

$$\tau_p = \sigma_n * \tan\left\{\varphi_b + \frac{4 * A_0 * \theta_{max}^*}{C + 1} \left[1 + \exp\left(-\frac{1}{9A_0} * \frac{\theta_{max}^*}{C + 1} * \frac{\sigma_n}{\sigma_t}\right) \right] \right\} \quad (8)$$

With the further use of the laser scanner technology, new criteria were developed (Tang 2014). The proposed shear strength criterion (T14) is capable of predicting the shear strength of rough joints.

$$\tau_p = \sigma_n * \tan\left[\varphi_b + 10 * \frac{A_0 * \theta_{max}^*}{(C + 1)} * \frac{(\sigma_t/\sigma_n)}{1 + (\sigma_t/\sigma_n)}\right] \quad (9)$$

All criteria are the common parameters A_0 , θ_{max}^* and C , proposed by Grasselli.

All of these criteria are very similar to each other, as they are written according to the equation (7), except for the criterion G01 (equation 6). Common to all of them is φ_b , which is multiplied by the tangent of base friction angle to which an additional angle is added, which represents the crushing of the rock teeth of the surface sample and also influence of dilation. The differences between the criteria are actually at this additional angle. It is described with the parameters of the surface, A_0 , C , θ_{max}^* .

Test procedure

Samples for direct shear test were taken from different rock formations from the northern part of Slovenia. The types of rock vary from clayey limestone, siltstone to the permo-carboniferous shale rock with very low geomechanical characteristics.

Among many samples, 19 of them were selected for testing. All samples have a natural fracture. The joint surface was scanned by 3D scanner system before the shear test to measure the morphology of the surface (fig. 3). A data processing programme was used to calculate the 3D statistical characterisation parameters.

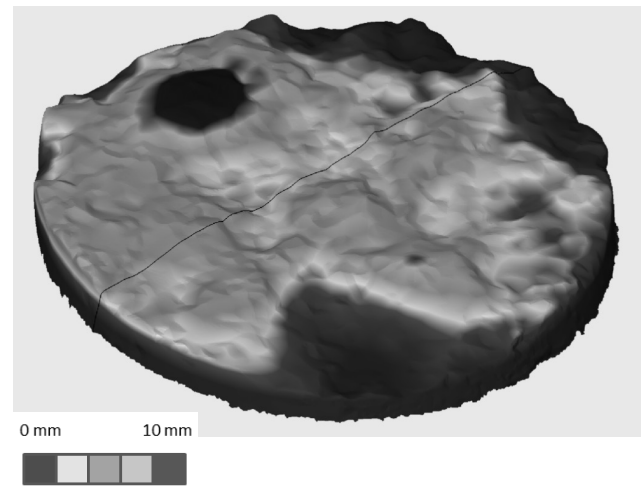


Fig. 3. An example of the scanned sample.

For the direct shear testing, the Robertson apparatus was used. This equipment is very useful for small samples which were acquired with borehole drilling. Shear tests of several rock joint samples under different normal loads have been tested (0.1, 0.2, 0.4 MPa) in order to relate the peak shear strength of a rock joint with the three-dimensional (3D) surface. The test was performed according to the standard ASTM D5607 -08. When the shear displacement reached the post-peak stage and stabilised for a while, the test was stopped. During shearing, normal deformation, horizontal deformation, normal load and shear force of the joint samples were monitored and recorded.

Results

The peak shear strength was calculated for every sample according to the criterion described in the previous chapter. The input data for the calculation are presented in Table 2 and include the input data for the shear peak strength criteria; G01, G06, X13 and T14. The results of the direct shear test and the results of the calculated peak shear strength according to the different criteria are presented in Table 3.

Table 3. Results of peak shear strength calculation under different criteria.

No.	τ_p measured (MPa)	τ_p G01 (MPa)	τ_p G06 (MPa)	τ_p X13 (MPa)	τ_p T14 (MPa)	τ_p X13 mod (MPa)
1	0.401	0.444	0.595	0.459	0.492	0.012
2	0.106	0.087	0.118	0.106	0.126	0.006
3	0.070	0.081	0.119	0.063	0.069	0.169
4	0.064	0.086	0.104	0.061	0.066	0.046
5	0.245	0.327	0.388	0.257	0.269	0.021
6	0.232	0.323	0.410	0.302	0.322	0.165
7	0.115	0.129	0.200	0.138	0.162	0.064
8	0.168	0.130	0.181	0.175	0.177	0.070
9	0.087	0.075	0.098	0.077	0.081	0.225
10	0.276	0.222	0.286	0.267	0.251	0.055
11	0.217	0.280	0.429	0.271	0.307	0.181
12	0.087	0.078	0.102	0.081	0.085	0.056
13	0.278	0.191	0.326	0.357	0.353	0.366
14	0.065	0.085	0.101	0.064	0.068	0.052
15	0.077	0.077	0.092	0.064	0.066	0.239
16	0.219	0.235	0.277	0.232	0.221	0.035
17	0.106	0.067	0.119	0.124	0.156	0.045
18	0.170	0.215	0.309	0.279	0.340	0.411
19	0.328	0.480	0.688	0.465	0.519	0.306

Table 2. Input data for the peak shear strength calculation.

No.	Lithology	σ_t (MPa)	σ_n (MPa)	ϕ_b (°)	A0 (-)	C (-)	Θ^*_{max} (°)
1	marly limestone	1.99	0.60	24	0.415	17.03	86.86
2	marly limestone	1.99	0.10	24	0.579	16.99	89.69
3	marly limestone	1.99	0.10	24	0.177	12.87	86.20
4	marly limestone	1.99	0.10	24	0.300	22.04	75.11
5	marly limestone	1.99	0.40	24	0.395	27.92	86.95
6	marly limestone	1.99	0.40	24	0.454	12.20	51.89
7	dolomite	2.17	0.15	25	0.341	11.90	90.00
8	perm. slate	0.30	0.20	24	0.542	15.04	86.21
9	perm. slate	0.30	0.10	24	0.472	9.22	42.93
10	perm. slate	0.30	0.40	24	0.471	9.30	41.42
11	siltstone	2.00	0.40	26	0.200	11.61	87.28
12	claystone	0.30	0.20	20	0.120	19.98	84.74
13	claystone	0.30	0.40	24	0.502	9.40	84.24
14	claystone	1.00	0.10	24	0.366	28.62	89.90
15	claystone	0.30	0.10	24	0.395	24.40	79.72
16	claystone	0.30	0.40	24	0.395	25.60	77.60
17	claystone	0.30	0.10	24	0.511	9.25	89.05
18	siltstone	2.00	0.20	30	0.515	12.46	84.87
19	dolomite	2.49	0.60	28	0.260	12.72	84.31

For all results, the average estimation error was calculated E_{ave} (Kulatilake et al., 1995), which is presented in Table 4.

$$E_{ave} = \frac{1}{m} \sum_{i=1}^m \left| \frac{\tau_{test} - \tau_{cal.}}{\tau_{test}} \right| * 100\% \quad (10)$$

Table 4. Average estimation error for every criterion.

τ_p	τ_p	τ_p	τ_p	τ_p
G01	G06	X13	T14	X13 mod (%)
(%)	(%)	(%)	(%)	
23	45	16	23	13

According to the results, a small correction was used to get a better correlation between the measured and calculated peak shear strength for the criterion X13. For the testing with the Roberson apparatus, the samples have to be smaller and with the small change of the equation, better correlation was achieved and modified criterion (X13mod) is presented in eq 11.

$$\tau_p = \sigma_n * \tan \left\{ \varphi_b + \frac{4,9 * A_0 * \theta_{max}^*}{C + 1} \left[1 + \exp \left(-\frac{1}{A_0} * \frac{\theta_{max}^*}{C + 1} * \frac{\sigma_n}{\sigma_t} \right) \right] \right\} \quad (11)$$

Discussion

This paper presented a detailed methodology to evaluate the three-dimensional roughness of joint surfaces in rock material. The presented methodology uses 3D surface measurements, which are becoming more widely available with the increasing availability of commercial opti-

cal measuring devices. The advantage of using 3D scanner is in determining the morphological parameters for the whole surface, not only on a single profile. The use of these parameters allows studying the directional micro-mechanical response of the entire sheared joint.

The proposed roughness evaluation methodology was demonstrated by digitizing and analysing the fracture surfaces of 19 specimens. Samples used in the referred studies (Grasselli, 2001, 2006; Xia, 2013; Tang, 2014) have a dimension at least 200 mm × 100 mm × 100 mm and were consolidated under high normal stresses (more than 1 MPa). It is often not possible to get large samples for testing material in a large direct shear test. Borehole samples are smaller and have various shapes and sizes. In this case, samples are usually tested in Robertson direct shear test apparatus.

In our case, the samples were taken from boreholes and were different with regard to their dimension and shape. The samples were tested under low normal load (no more than 0.4 MPa). The peak shear strengths of natural joints obtained experimentally in laboratory tests were compared with the values calculated by Eq. 7, 8, 9 and 10 as listed in Table 3. According to the correlation analysis, the calculated values are slightly larger than the measured values (fig. 4), but the predicted shear strength from criteria is close to the experimental shear strength of natural joints. Hence, it can be deduced that the proposed shear strength criterion is capable of predicting the shear strength of rough joints. For all criteria, the average estimation errors were

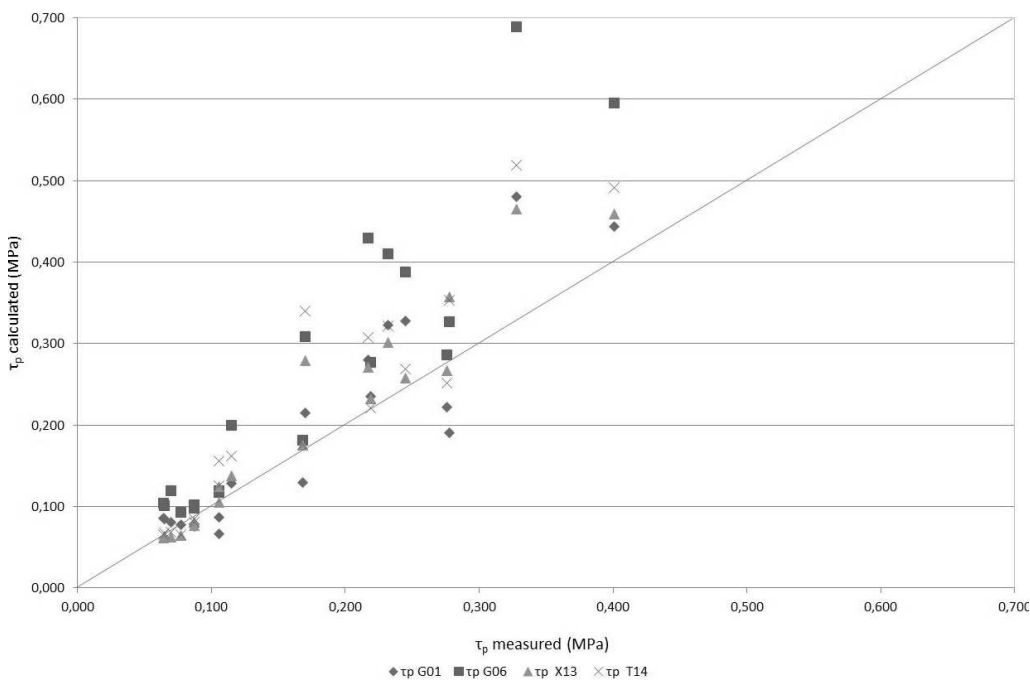


Fig. 4. The comparison between calculated.

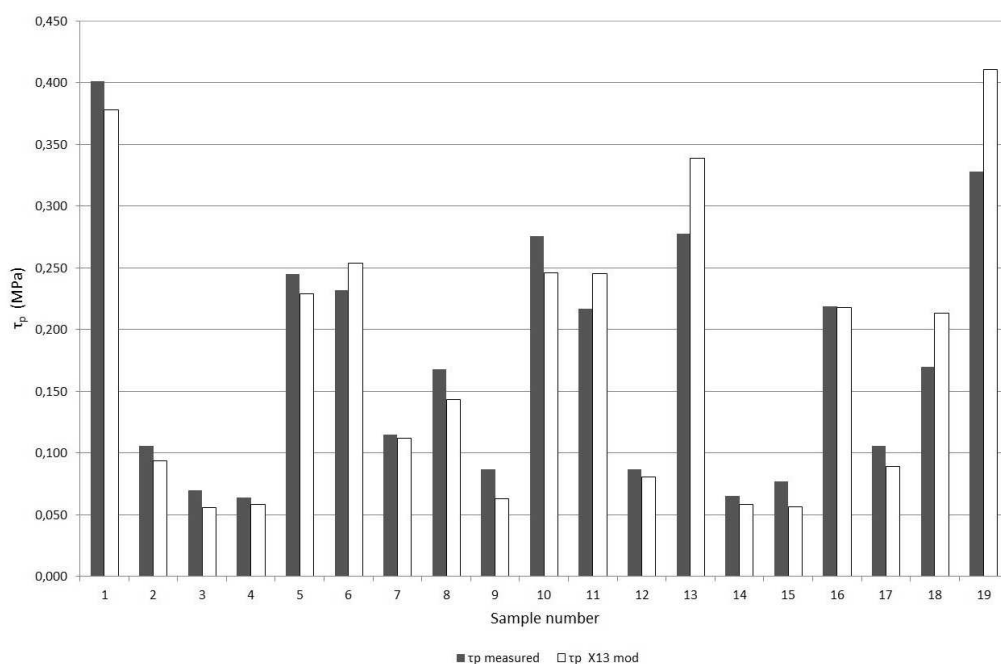


Fig. 5. The comparison between calculated and measured peak shear strength for modified criterion X13mod.

calculated according to the eq. 10. The criterion which fits the best with the measurement data is criterion X-13 with the average estimation error 16 %. To improve the results, we changed the criterion X-13 in a part of the equation where the area A_0 is included (Eq. 11), because smaller samples in our research were used. With this small correction, the average estimation error decreases to 13 %. The comparison between calculated and measured peak shear strength for modified criterion X13mod is presented in fig. 5.

For future research, it is necessary to test more samples of the same lithology. The samples tested in our case were very different in the sense of the surface roughness. We could probably get better results if tests were done for every type of rock separately, of course with an adequate number of samples. The size of the samples affected the results and there is probably a reason that the average estimation errors in our research work were not lower for other already known and used criteria.

Conclusions

Shear behaviour of rock joints is investigated with the Robertson apparatus. The shear strength increases with the increasing of normal stress and roughness. The proposed modified criterion can be used as a predictive tool to assess the peak shear strength under the low normal load and if we could only use small samples from the investigation boreholes.

Development of the scanning technology could be used for high-resolution surface characterization in the laboratory, but may also allow joint characterisation in-situ in future research.

Acknowledgement

This paper was prepared from the some results of work carried out within the research project "DESTinationRAIL - Decision Support Tool for Rail Infrastructure Managers", No. 636285 funded by the European Union, within the scope of HORIZON 2020.

References

- Barton, N. 1973: Review of a new shear-strength criterion for rock joints. *Eng. Geol.*, 7/4: 287–332.
- Barton, N. 1976: The shear strength of rock and rock joints. *Int. J. Rock. Mech. Min. Sci. Geomech. Abstr.* 13/9:255–279.
- Barton, N. & Choubey, V. 1977: The shear strength of rock joints in theory and practice. *Rock Mech* 10/1–2:1–54.
- Fifer, B.K. 2010: Determining the Surface Roughness Coefficient by 3DScanner. *Geologija*, 53/2: 147–152, doi: <https://doi.org/10.5474/geologija.2010.012>.
- Grasselli, G. 2001: Shear strength of rock joints based on quantified surface description. PhD Dissertation. Ecole Polytechnique Federale de Lausanne, Lausanne: 126 p..
- Grasselli, G. 2006: Shear Strength of Rock Joints Based on Quantified Surface Description. *Rock Mechanics and Rock Engineering*, 39/ 4: 295–314, doi: <https://doi.org/10.1007/s00603-006-0100-0>.
- Grasselli, G. & Egger, P. 2003: Constitutive law for the shear strength of rock joints based on three-dimensional surface parameters. *International Journal of Rock Mechanics and Mining Sciences*, 40/1: 25–40, doi: [https://doi.org/10.1016/S1365-1609\(02\)00101-6](https://doi.org/10.1016/S1365-1609(02)00101-6).

Grasselli, G., Wirth, J. & Egger, P. 2002: Quantitative three-dimensional description of a rough surface and parameter evolution with shearing. *International Journal of Rock Mechanics and Mining Sciences*, 39/6: 789–800, doi: [https://doi.org/10.1016/S1365-1609\(02\)00070-9](https://doi.org/10.1016/S1365-1609(02)00070-9).

Hoek, E. 2000: Shear strength of discontinuities. *Rock engineering*, 61–72, <http://www.rock-science.com/hoek/PracticalRockEngineering.asp>.

Hoek, E. & Bray, J. W. 1981: *Rock slope engineering*. Vancouver, CRC Press.

Hoek, E. & Brown, E. T. 1980: Empirical strength criterion for rock masses. *Journal of Geotechnical and Geoenvironmental Engineering*, 106: 1013–1035.

Hong, E. S., Lee, J. S. & Lee, I. M. 2008: Underestimation of roughness in rough rock joints. *International Journal for Numerical and Analytical Methods in Geomechanics*, 32: 1385–1403, doi: <https://doi.org/10.1002/nag.678>.

Huang, S. L., Oelfke, S. M. & Speck, R. C. 1992: Applicability of fractal characterization and modelling to rock joint profiles. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 29: 89–98.

Kulatilake, P. H. S. W., Balasingam, P., Park, J. & Morgan, R. 2006: Natural rock joint roughness quantification through fractal techniques. *Geotechnical & Geological Engineering*, 24: 1181–1202, doi: <https://doi.org/10.1007/s10706-005-1219-6>.

Kulatilake P. H. S. W., Shou G., Huang T. H., Morgan R. M. 1995: New peak shear strength criteria for anisotropic rock joints. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 32/7: 673–697, doi: [https://doi.org/10.1016/0148-9062\(95\)00022-9](https://doi.org/10.1016/0148-9062(95)00022-9).

Lee, H. S. & Ahn, K. W. 2004: A prototype of digital photogrammetric algorithm for estimating roughness of rock surface. *Geosciences*, 8/3: 333–341.

Lee, H. S., Park, Y. J., Cho, T. F. & You, K. H. 2001: Influence of asperity degradation on

the mechanical behaviour of rough rock joints under cyclic shear loading. *International Journal of Rock Mechanics and Mining Sciences*, 38/7: 967–980, doi: [https://doi.org/10.1016/S1365-1609\(01\)00060-0](https://doi.org/10.1016/S1365-1609(01)00060-0).

Odling, N. E. 1994: Natural fracture profiles, fractal dimension and joint roughness coefficients. *Rock Mechanics and Rock Engineering*, 27: 135–153.

Patton, F.D. 1966: Multiple modes of shear failure in rock. In: *Proceedings of the 1st congress of International Society for Rock Mechanics*, 1: 509–513.

Pellet, F. L., Keshavarz, M. & Boulon, M. 2013: Influence of humidity conditions on shear strength of clay rock discontinuities. *Engineering Geology*, 157: 33–38.

Reeves, M. J. 1985: Rock surface roughness and frictional strength. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 22/6: 429–442, doi: [https://doi.org/10.1016/0148-9062\(85\)90007-5](https://doi.org/10.1016/0148-9062(85)90007-5).

Reich, C., Ritter, R. & Thesing, J. 2000: 3-D shape measurement of complex objects by combining photogrammetry and fringe projection. *Optical Engineering*, 39/1: 224–231, doi: <https://doi.org/10.1117/1.602356>.

Tang, Z. C., Liu, Q. S. & Huang, J. H. 2014: New criterion for rock joints based on three-dimensional roughness parameters. *Journal of Central South University*, 21/12: 4653–4659, doi: <https://doi.org/10.1007/s11771-014-2473-7>.

Tang, Z. C. & Wong, L. N. Y. 2015: New Criterion for Evaluating the Peak Shear Strength of Rock Joints Under Different Contact States. *Rock Mechanics and Rock Engineering*, 49/4: 1191–1199, doi: <https://doi.org/10.1007/s00603-015-0811-1>.

Xia, C. C., Tang Z. C., Xiao W. M. & Song, Y.L. 2013: New Peak Shear Strength Criterion of Rock Joints Based on Quantified Surface Description. *Rock Mechanics and Rock Engineering*, 47/2: 387–400, doi: <https://doi.org/10.1007/s00603-013-0395-6>.