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Theorems and laws defining the distribution of the geological random variable

Teoreme i zakoni koji definišu raspodelu geološke slučajne promenljive

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Abstract

This paper treats the theoretical basis of distribution of population of chemical elements (atoms) in a geological body according to 6 theorems and 3 laws of distribution. The mathematical background of these theorems and laws of distributions of geological random variables are well beyond the framework of this paper; this material has been presented in a paper at the 27th International Geological Congress, Moscow, on August 6, 1984.

By the dual nature of a sample as a unit magnitude the existence of two geological populations is introduced within a statistical sample that represents the elementary population of the investigated chemical element within a geological body. This finding represents an important discovery that modifies the methodology of statistical investigations in geology (geochemistry). This author has directed his efforts to further development of this new methodology of statistical study of distribution of chemical elements in geological bodies.

Kratak izvod

U ovom radu je data teoretska osnova raspodele populacije hemijskih elemenata (atoma) u geološkom telu pomoću 6 teorema i 3 zakona raspodele. Matematička osnovica ovih teorema i zakona raspodele geološke slučajne promenljive prevazilazi obim ovog rada, koji predstavlja referat izložen na 27. Međunarodnom geološkom kongresu u Moskvi dana 6. avgusta 1984. godine, te je zato izostavljena.

Dvojna priroda probe kao jedinične veličine uslovljava postojanje dve geološke populacije izučavanog hemijskog elementa u geološkom telu. Ovo saznanje predstavlja otkriće, a ono suštinski menja metodologiju statističkih ispitivanja u geologiji (geohemiji). Napori autora su usmereni na razradu ove nove metodologije statističkih izučavanja raspodele hemijskih elemenata u geološkim telima.

From the geochemical point of view, atoms of chemical elements are the basic building elements of geological bodies of the Earth's crust. The concept of an atom is identical to the concept of a chemical element (Vernadskiy, 1983). The geological population of a chemical element in a geological body is, in its essence, the population of atoms of a certain kind. This population is infinitely large, continuous and uncountable even in relatively small geological bodies or their parts (samples).

Geological population of atoms of a chemical element is a natural phenomenon of statistical character and it obeys the law of large numbers. The atoms react, thus producing minerals which constitute geological bodies. The distribution of the geological population of chemical elements is indirectly connected to the distribution of minerals in a geological body.

Redistribution of the population of atoms in minerals has the character of separation in the sense that considerable differences in concentrations of chemical elements are made on the level of elementary volumes (these volumes correspond, in dimension, to mineral grains in rocks).

The basic geological population of atoms of a chemical element in a geological body cannot be investigated directly. Therefore sampling methods have been developed as a means of practical investigation of geological bodies. Sampling is an experiment which reveals concentrations of elements in a sample; these concentrations are statistically reliable events (the content of an element is always realized, taking into account also the conditional "zero", below the threshold of the analytical method).

A sample being the material representative of a geological body constitutes a unit value, while the set of samples represents the basic geological population of the element, i. e., it is the statistical sample of a geological body. The characteristics of a geological population in a statistical sample are attributed to the basic geological population of an element in a geological body.

The distribution of a chemical element in a geological body is defined by means of a statistical sample in 6 theorems and 3 distribution laws. These theorems and laws will be presented and commented in this paper. The theoretical basis of these theorems and laws lies beyond the scope of this paper; it has been presented in previous publications (Omaljev, 1977, 1978, 1982a).

Theorem 1: A sample, being the material representative of a geological body, has two characteristics: 1) the unit mass of the sample material (q), and 2) the unit mass of the investigated chemical element (X), shown by concentration (x) as a measure of the contents of its atoms; this represents the geological information.

The sample material consists of the atoms of all present elements, therefore the sum of atomic masses makes the unit mass of the material of a sample (q). The unit mass of an investigated element (X) consists of its atoms and it is a part of the mass contained within the sample matter. The unit mass of a sample (q) represents the "wrapping" for the unit mass of the investigated chemical element (X). This fact is of extreme importance, and it is the basis of the new methodology of statistical investigation of the geological population distribution. The unit mass (of samples) represents the "quantum" in statistical investigations; it can be accepted or rejected.

The concentration of a chemical element (x) in a sample, being the measure of the chemical element's contents in the mass of the sample, is represented as follows:

$$x = \frac{X}{q}$$

and for the unit mass equal to unity (q = 1), it is

$$x = X$$

i. e. this is unit concentration, and it is numerically equal to the unit mass of an investigated chemical element (X).

In this way, the unit concentration (x) unites the two characteristics of the unit volume of the sample (v): 1) the unit mass of a sample (q), and 2) the unit mass of an investigated chemical element (X); these is information about a geological population according to characteristics of (q) and (X).

A set of samples of which consists the statistical sample is also a geological population. This geological population is related to the sample's characteristics; i. e. the statistical sample contains two geological populations: 1) the population of samples as unit masses of material (q), and 2) the population of unit masses of an investigated chemical element (X).

The statistical sample as a geological population is of discrete (corpuscular) character. The number of data is finite and countable. On the other hand, the basic geological population in a geological body is infinitely large, continuous and uncountable. In order to overcome the discrete nature of a statistical sample, which has to be fulfilled in order to represent the basic geological population in a geological body, an infinitely large number of data is required.

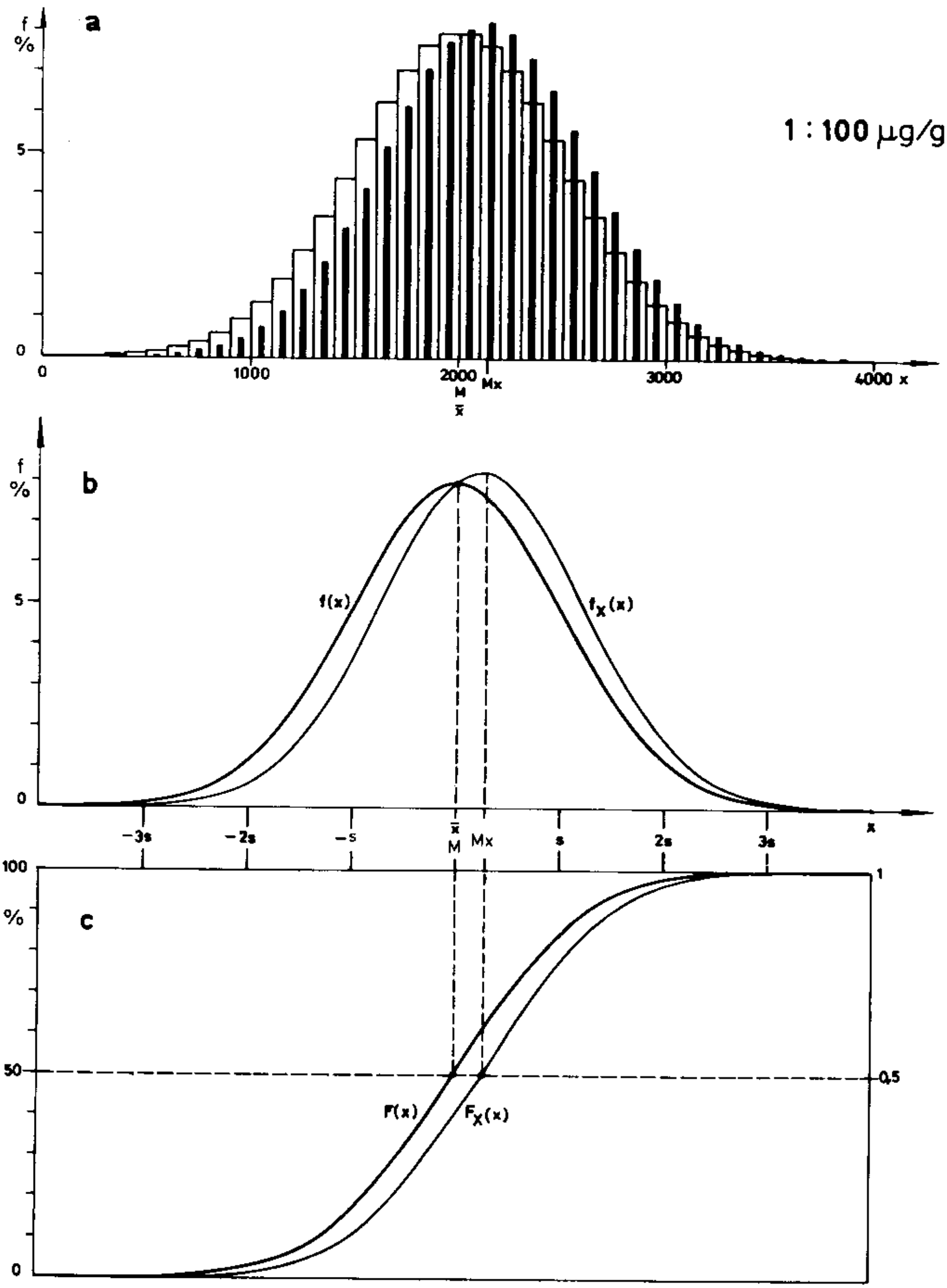
Depending on the thereshold of sensitivity of the statistical investigation, it is possible to define a conditionally large number of data in a statistical sample; this infinitely large number is defined when the relative frequency of the unit mass of a sample (q) becomes, conditionally, equal to zero. For example:

Relative frequency of the unit mass (q)	Number of data in the statistical sample
$\frac{q}{\sum q} 100 = \frac{1}{N} 100$ (%)	N
0 %	> 200
0,0 %	> 2,000
0,00 %	> 20,000

etc.

Theorem 2: Unit masses of sample material (q) and unit masses of an investigated chemical element (X) are functions of concentration (x); they are represented by symbols f(x) and $f_X(x)$.

Graphically, the illustration of these functions can be a dual polygon, a dual histogram, or a dual diagram (fig. 1, 2, 4 and 5). Concentration (x), as an independent geological variable, is plotted on the abscissa, while the dependent variables, f(x) and $f_X(x)$, are plotted on the ordinate.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The dual nature of polygons, histograms and diagrams is derived from the fact that two geological populations exist within a statistical sample. The pillars of dual histograms (fig. 1, 2, 4 and 5) represent the intensities of the function f(x) and they contain pillars which represent the function $f_X(x)$. However, it is only a graphical representation, since both widths are the same on the abscissa. These functions are:

f(x) — The relative frequency distribution of the sample's material unit masses (q) versus the independent geological variable (x).

$f_X(x)$ — The relative frequency distribution of the investigated chemical element's unit masses in the sample's material (X) versus the independent geological variable (x).

The quotient of concentrations of chemical elements in a sample (Na/K, Th/U) has also the property of an independent geological variable.

If more concentrations of chemical elements are investigated in a sample (multicomponent sample analysis), then all investigated elements (components) have their own geological populations. Unit masses of each of the elements behave like functions (dependent geological variables) of concentrations of each of the investigated chemical elements.

In a two-component system there are 6 functions (unit mass of a sample q, and each of the two investigated elements X and Y is a function of two concentrations x and y), while in three-component systems there are 12 functions, etc. In multi-component systems the number of functions grows rapidly, and therefore the investigation of these systems becomes extremely complicated.

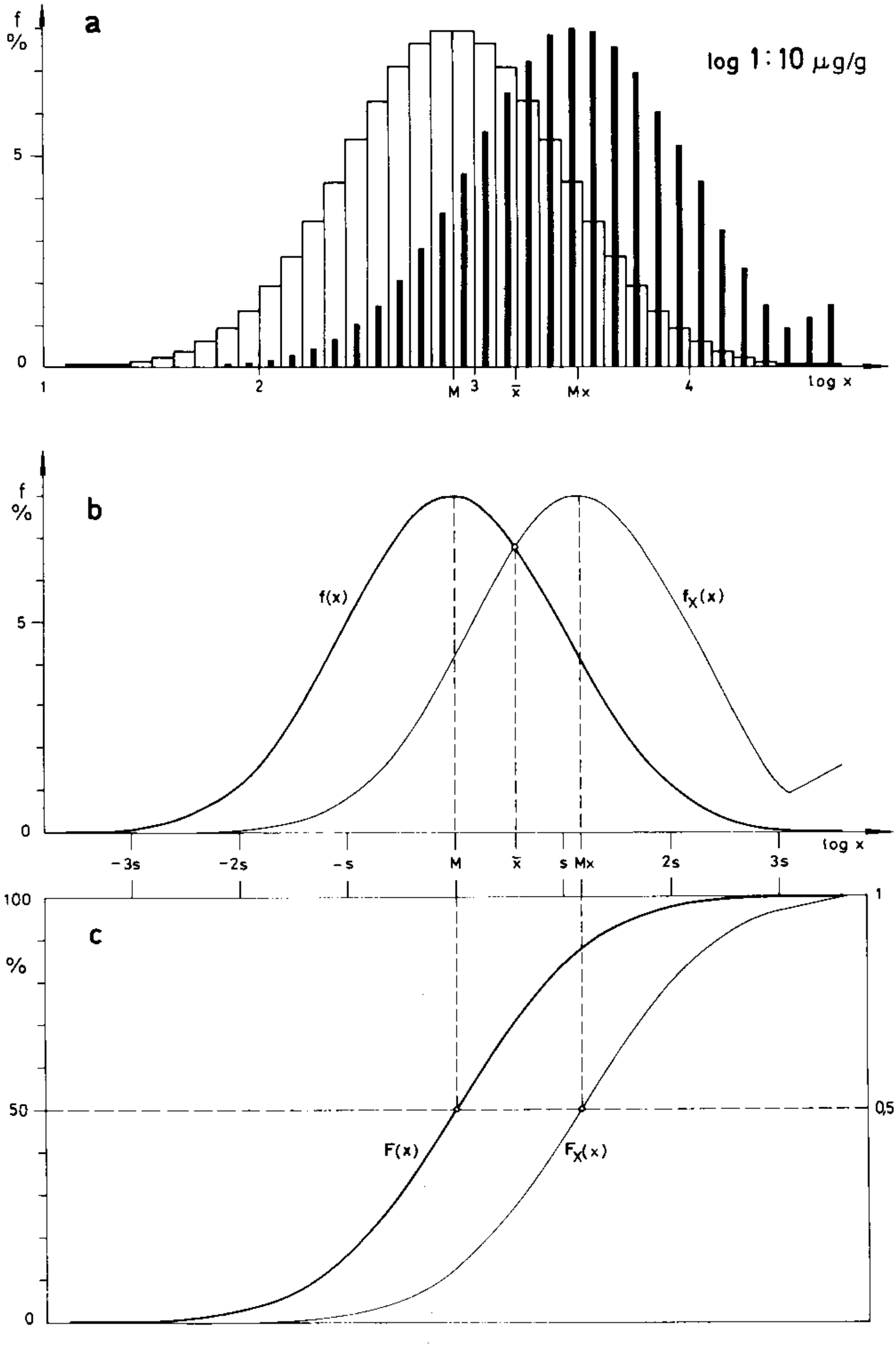
Theorem 3: The statistical scale is a standardization measure of the independent geological variable (x); it defines the position and size of each of the classes and isolines of concentration (geochemical equidistance) of an investigated chemical element on the abscissa of the histogram (diagram) and on the geochemical map.

The relief of the geochemical field, according to the new definition given by the author (Omaljev, 1982 b, 1983), is basically similar to the relief of Earth's surface. The graphical methods of topography and of geochemical field representation have to be, basically, the same; both surfaces, are presented by isolines of potential on geochemical maps: 1) elevation for topography, and 2) concentration intensities of an investigated chemical element for the relief of the geochemical. Of course, the equidistance standardization has to be done according to the same principles.

Geochemical equidistance is a number which shows the interval between adjacent concentration isolines; its standardization is called the »statistical scale«.

Fig. 1. The normal (symmetrical) distribution of geological random variable
a) Dual histogram of distribution of functions f(x) (pillars) and $f_X(x)$ (full pillars)
b) Density curves of distribution of functions f(x) and $f_X(x)$
c) Commulative curves of distribution of functions F(x) and $F_X(x)$

Sl. 1. Normalna (simetrična) raspodela geološke slučajne promenljive
a) Dvojni histrogram raspodele funkcija f(x) (stubovi) i $f_X(x)$ (puni stubići)
b) Krive gustine raspodele funkcija f(x) i $f_X(x)$
c) Kumulativne krive raspodele funkcija F(x) i $F_X(x)$



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c
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%
50
0
1
0,5
F(x)
$F_X(x)$

The linear statistical scale is the ratio of unity to the geochemical equidistance, and it is presented in the usualy way (1 : 1, 1 : 2, 1 : 5, 1 : 10, 1 : 100, etc.). Geochemical equidistance is the interval of concentration variation of each of the classes on the abscissa of dual histograms. It is essential that the value "zero", for concentration, is placed at the coordinate origin.

The logarithmic statistical scale is defined in a similar way: it is the ratio of unity to the geochemical equidistance in logarithmic relations. The logarithmic geochemical equidistance is represented by equal segments on the logarithmic abscissa (log x — fig. 2). As we move the decimal point, the whole relative numbers of logarithms correspond to decades of natural numbers; that is what we call "logarithmic decades". Logarithmic decades are equal segments on the logarithmic abscissa (log x). Logarithmic geochemical equidistance must contain the whole logarithmic decade or its whole number quotient.

Logarithmic statistical scale is denoted by "log 1 : 1" (the whole logarithmic decade), "log 1 : 5 (1/5 of the logarithmic decade), "log 1 : 10" (logarithmic decade divied in 10 parts — fig. 2), etc.

Theorem 4: The distribution of the geological random variable is defined by three parameters: 1) mathematical expectation (μ) of the population (arithmetical mean $\bar{x}$ of the statistical sample), 2) median (M), and 3) median of the investigated chemical element (Mx). The medians divide geological population in two halves (the depleted one and the enriched one) according to the mass of material (q) and according to the mass of an investigated chemical element (X).

Theorem 5: Mathematical expectation (μ) of the population (arithmetical mean $\bar{x}$ of the statistical sample) is the common point (the intersection point) of the density curves f(x) and $f_X(x)$ and it is the point of equalization of unit relative frequencies ($X_s/\Sigma X = q/\Sigma q = 1/N$) of these functions.

The mean contents. of a chemical element in a geological body is the mean value E(x) of the geological random variable, and it represents mean concentration of the atoms of an investigated chemical element. This value is unknown, and there is no method for accurate determination of mathematical expectation. For this reason arithmetical mean is introduced as an estimator of the mathematical expectation, being the mean contents.

Mathematical expectation (arithmetical mean), graphically taken, represents the intersection point (fig. 1, 2, 4 and 5) of the density curves f(x) and $f_X(x)$; this points corresponds to the equalization point of relative frequencies of unit masses (q) and (X) of these functions ($X_s/\Sigma = q/\Sigma q = 1/N$). The value of mathematical expectation (arithmetical mean) is equal to the ratio of areas under the density curves $f_X(x)$ and f(x):

Fig. 2. The lognormal distribution of geological random variable
a) Dual histogram of distribution of functions f(x) (pillars) and $f_X(x)$ (full pillars)
b) Density curves of distribution of functions f(x) and $f_X(x)$
c) Cummulative curves of distribution of functions F(x) and $F_X(x)$

Sl. 2. Lognormalna raspodela geološke slučajne promenljive
a) Dvojni histogram raspodele funkcija f(x) (stubovi) i $f_X(x)$ (puni stubići)
b) Krive gustina raspodele funkcija f(x) i $f_X(x)$
c) Kumulativne krive raspodele funkcija F(x) i $F_X(x)$

$$\mu = \frac{\int_a^b f_X(x)\,dx}{\int_a^b f(x)\,dx} \quad ; \quad \bar{x} = \frac{\Sigma X}{\Sigma q} = \frac{\Sigma X}{N} = \frac{\Sigma x}{N}$$

Median (M) is a numerical value of the concentration of the investigated chemical element of the central member of the statistical sample population. It represents the grouping central of a geological population of the statistical sample, but it does not have the character of the mean value E(x).

The population of unit masses of an investigated chemical element (X), in a statistical sample, has its own median of an investigated element (Mx). It is the middle member of the set which divides the population in two halves so that each of them contains, approximately, one half of the atoms, i. e. of the mass of an investigated chemical element.

Function $f_X(x)$ and median Mx are new in statistics (Omaljev, 1977, 1978, 1982a).

Moduses (Mo and Mo_X) are the most frequent members of the set of samples and they are positioned at the maximus of density curves f(x) and $f_X(x)$. Complex geological populations consist of several partial populations which have their own moduses. Modus is not the characteristics parameter of the distribution, like the mean contents ($\bar{x}$) and medians (M and Mx), as defined by Theorem 4.

Theorem 6: The position of a geological population on the abscissa of the independent geological variable is defined by the interval of concentration of a chemical element ($x_{max} - x_{min}$, being the element of a geochemical field) in the samples of the statistical sample. Outside of this interval the geological random variable is not defined.

The form of the density curve f(x) is strictly dependent on the character of the distribution; of the geological random variable; the determination of the distribution type is based on this fact. Five density curve types have been described (fig. 3) so far (Bogatskiy, 1963). This author is, however, of the opinion that there are only three types of the geological random variable distribution (Omaljev, 1978, 1982a):

1) The symmetrical distribution
2) The left-asymmetrical distribution
3) The right-asymmetrical distribution

The type of the geological random variable distribution is mathematically defined by the relation (position) of parameters which represent the grouping centre of the statistical sample's data. On the basis of Theorem 4 these are: 1) mathematical expectation of the population (arithmetical mean of the statistical sample), 2) median (M), and 3) median of an investigated chemical element (Mx).

The laws of the geological random variable distribution mathematically define the distribution type.

First law: The position of the mathematical expectation of the population (arithmetical mean of the statistical sample) in symmetrical distribution, approximatelly coincides on the abscissa with the median (M):

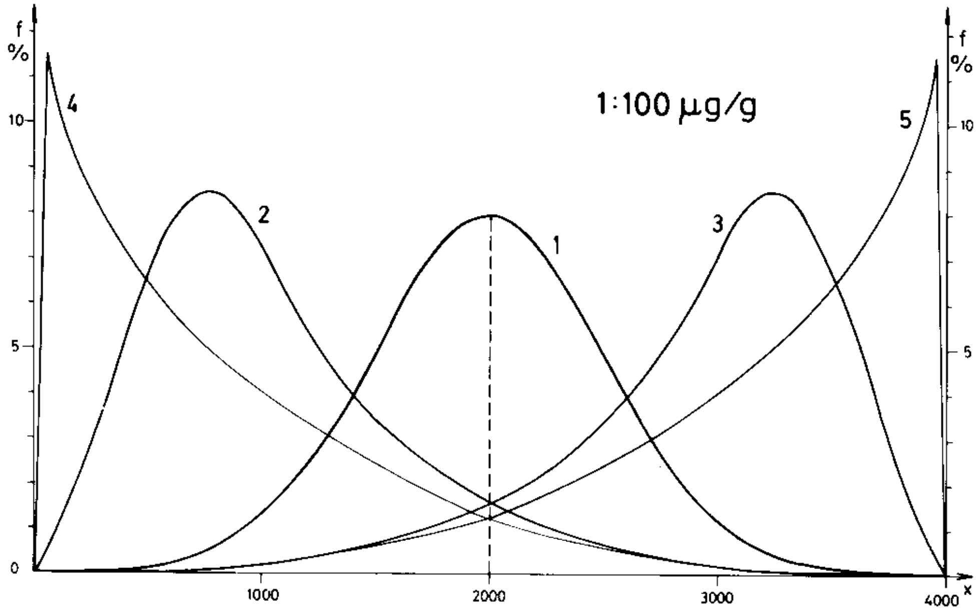


Fig. 3. The types of density curves of distribution of geological random variable
1 Normal (symmetrical) distribution, *2* Left-asymmetrical distribution, *3* Right-asymmetrical distribution, *4* Left-asymmetrical hyperbolic distribution, *5* Right-asymmetrical hyperbolic distribution

Sl. 3. Tipovi krivih gustina raspodele geološke slučajne promenljive
1 Normalna (simetrična) raspodela, *2* levoasimetrična raspodela, *3* desnoasimetrična raspodela, *4* levoasimetrična hiperbolična raspodela, *5* desnoasimetrična hiperbolična raspodela

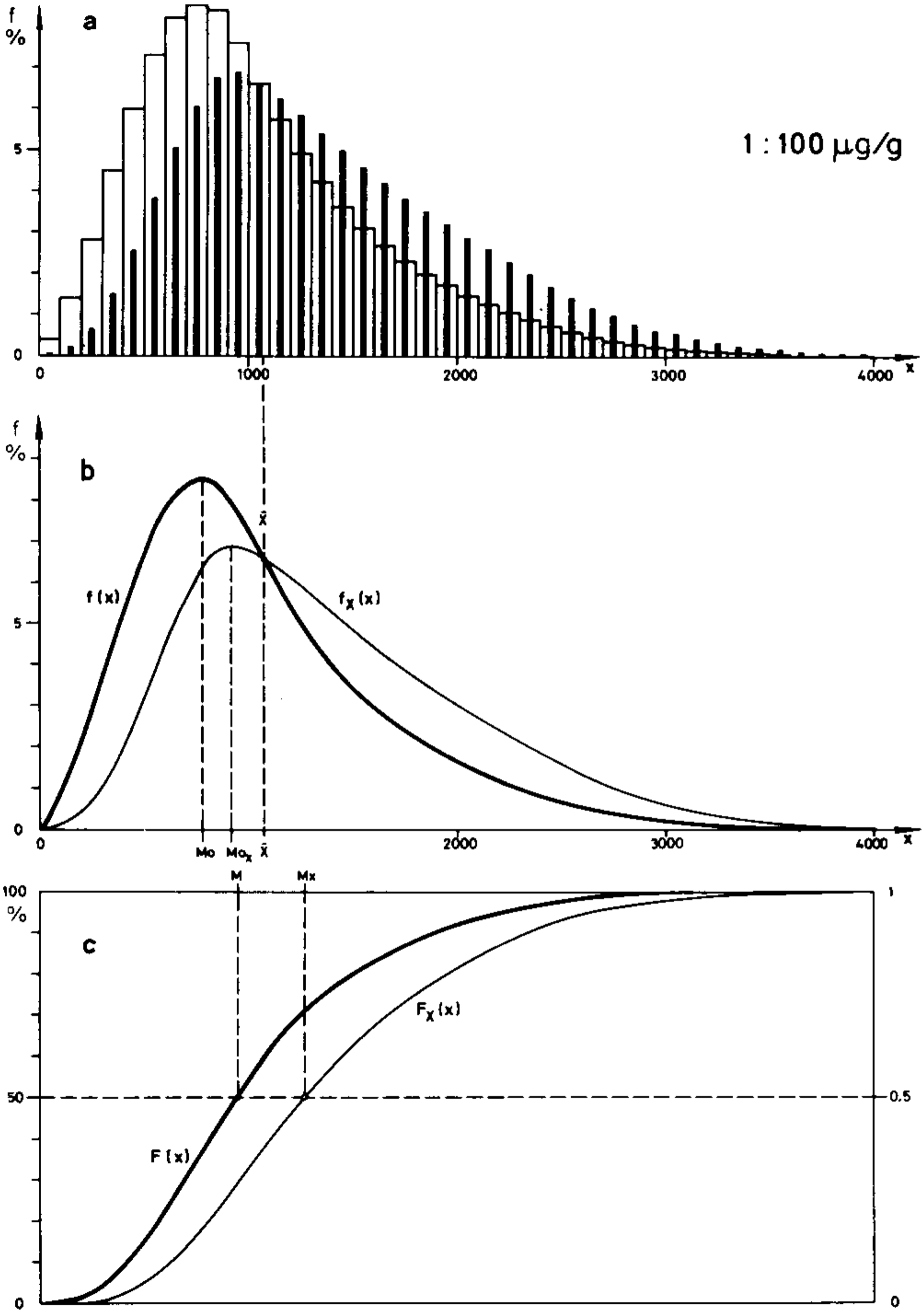
$$\bar{x} \approx M < Mx$$

Symmetry of the density curve f(x) is defined with respect to the median (M) as a result of Theorem 4. The normal distribution (fig. 1) is the special case of absolute coincidence of mathematical espectation (arithmetical mean) and median ($\bar{x} = M$); it represents the central ("zero") position within the spectrum of various density curve forms caused by the left (positive) or right (negative) asymmetry, or, in other words, by the different degrees of flatness.

Second law: The position of the mathematical expectation of the population (arithmetical mean of the statistical sample) on the abscissa, in the left-asymmetrical distribution (positive assymetry), is between the two medians:

$$M < \bar{x} < Mx$$

The left (positive) asymmetry (fig. 4) might be of various shapes, even hyperbolic in the extreme case. By application of the logarithmic statistical scale, the elongation of the curve to the right side is eliminated (logarithmic abscissa "compresses" the curve to the left side), and therefore the second law is also called the "logarithmic law".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Numerous authors refer to the logarithmic law as to the "lognormal law" of distribution (A h r e n s, 1963; R o d i o n o v, 1961; C a r l i e r, 1964; and many others). However, the lognormal density curve f(x) is only the special case when it becomes symmetrical with respect to its median M (fig. 2).

The same effect is gained if we apply the exponential relations on the abscissa (x^m), for m not equal to one (m — 1 is the linear relationship), and therefore the second law can be generalized as the "exponential law" of distribution. Logarithm is only a special form of exponential relations, so the logarithmic law is a special case of the exponential law of geological random variable distribution (the second law).

This problem has not been considered with full attention in literature, and that is the reason for different interpretations of analysis of the left-asymmetrical distribution of a geological random variable.

Third law: The position of the mathematical expectation of the population (arithmetical mean of the statistical sample) on the abscissa, in right-asymmetrical distribution (negative asymmetry), is to the left of medians:

$$\bar{x} < M < Mx$$

Right (negative) asymmetry (fig. 5) may take various forms, up to the hyperbolic density curve f(x). For this type of distribution a procedure for the elimination of elongation to the left side has not been found yet.

Some density curve forms, in the case of left-asymmetrical and right-asymmetrical distribution types, are symmetrical with respect to the normal distribution median. It is the result of the law of large numbers in the course of generation of basic geological populations. It also means that there is unity between the processes of accumulation of chemical elements in nature. On the basis of central limit theorem's assumptions all distributions show a tendency towards the symmetrical (normal) distribution, and this is the reason why it has the central position in the spectrum of different density curves forms (fig. 3).

The characteristics of the geological random variable distribution are mathematically defined by the distribution parameters (Theorem 4), while their mutual relations define the distribution type. Distribution parameters are unique for all types of density curves, while the distribution laws unify the treatment of these parameters. The application of exponential and logarithmic relations on the abscissa is used only to symmetrize the density curves f(x) with respect to their medians. Bearing this in mind it is not possible to use the geometrical or exponential mean to calculate the mean value E(x). According to Theorem 5, the mathematical expectation (arithmetical mean) is the only estimate for the mean value E(x).

Fig. 4. The ideal left-asymmetrical distribution (positive asymmetry) of geological random variable

a) Dual histogram of distribution of functions f(x) (pillars) and $f_X(x)$ (full pillars)

b) Density curves of distribution of functions f(x) and $f_X(x)$

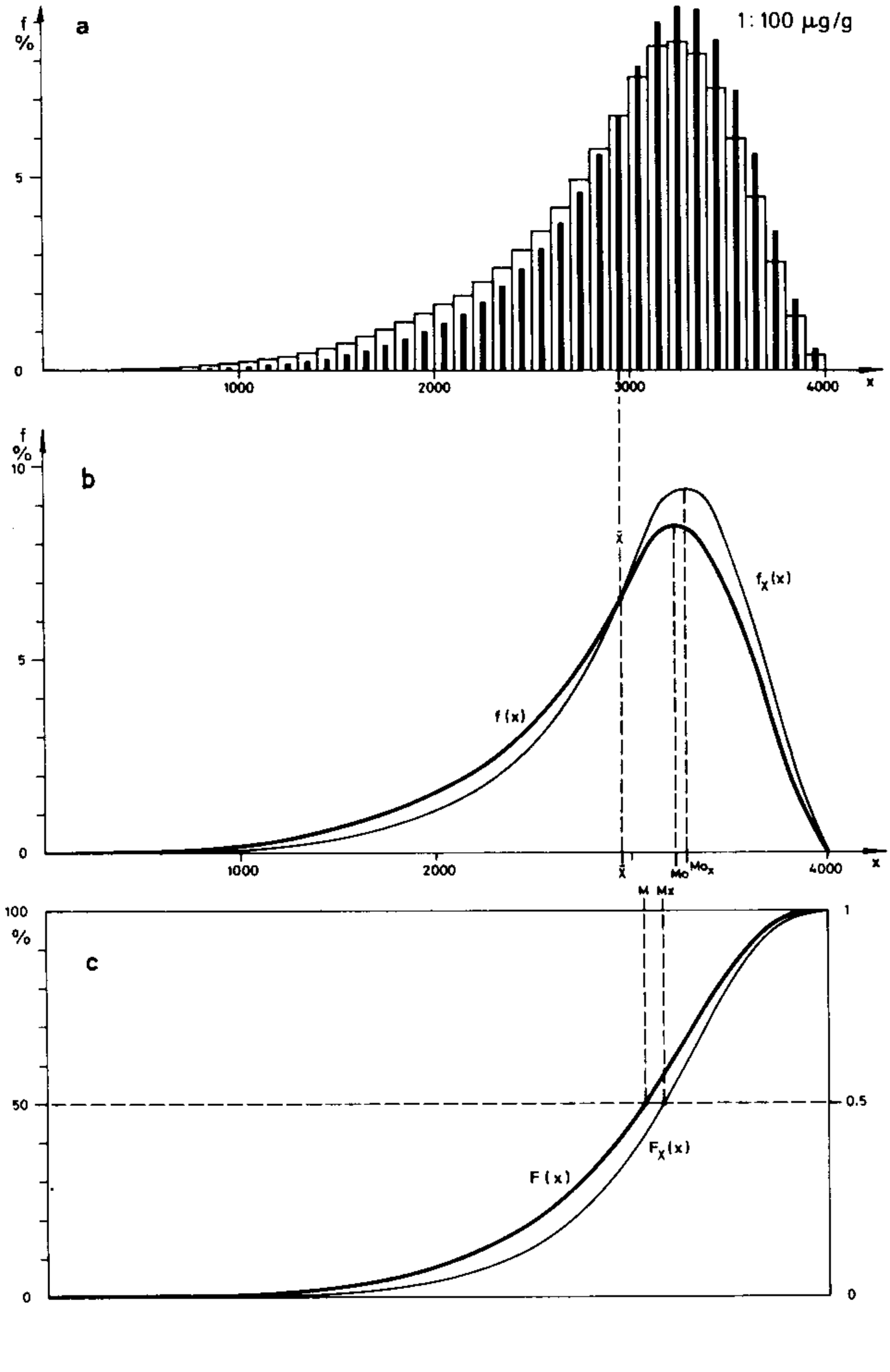
c) Cummulative curves of distribution of functions F(x) and $F_X(x)$

Sl. 4. Idealna levoasimetrična raspodela (pozitivna asimetrija) geološke slučajne promenljive

a) Dvojni histogram raspodele funkcija f(x) (stubovi) i $f_X(x)$ (puni stubići)

b) Krive gustine raspodele funkcija f(x) i $f_X(x)$

c) Kumulativne krive raspodele funkcija F(x) i $F_X(x)$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Teoreme i zakoni koji definišu raspodelu geološke slučajne promenljive

Izvod

Osnovni elementi građe geoloških tela zemljine kore, sa aspekta geohemije, su atomi hemijskih elemenata. Geološka populacija elemenata (atoma) u geološkom telu je beskonačno velika, neprekidna i neprebrojiva. Ova osnovna populacija nije dostupna direktnom opažanju i zato su razvijene metode oprobavanja. Proba predstavlja jediničnu veličinu, a skup proba čini statistički uzorak koji predstavlja osnovnu populaciju.

Raspodela elemenata (atoma) u geološkim telima je definisana sa **6 teorema** i 3 zakona raspodele. Teoretska osnova ovih teorema je ranije izložena u publikacijama (Omaljev, 1977, 1978, 1982a).

Teorema 1: Proba kao materijalni predstavnik geološkog tela ima dva obeležja: 1) jediničnu masu materijala probe (q) i 2) jediničnu masu ispitivanog hemijskog elementa (X) iskazanu koncentracijom (x) kao merom zastupljenosti njegovih atoma, što predstavlja geološku informaciju.

Koncentracija elementa se izražava odnosom $x = X/q$, a za jediničnu masu materijala probe ravnu jedinici ($q = 1$) dobijamo da je jedinična koncentracija brojno jednaka jediničnoj masi hemijskog elementa u probi ($x = X$).

Kao posledica teoreme 1 u statičkom uzorku postoje dve geološke populacije: 1) populacija proba kao jediničnih masa materijala (q) i 2) populacija jediničnih masa ispitivanog elementa (X) u probi.

Statistički uzorak, kao populacija, ima diskretan karakter; broj podataka je konačan i prebrojiv. Zavisno od praga osetljivosti, moguće je definisati uslovno beskonačno veliki broj podataka u statističkom uzorku; a to je broj podataka kada je relativna frekvencija jedinične mase probe uslovno jednaka nuli ($q/\Sigma q = 1/N = 0\ \%$).

Teorema 2: Jedinične mase materijala probe (q) i jedinične mase ispitivanog hemijskog elementa (X) su funkcije koncentracije (x); što predstavljamo simbolima $f(x)$ i $f_X(x)$.

$f(x)$ — Raspodela frekvencija jediničnih masa materijala probe (q) po nezavisnoj geološkoj promenljivoj (x).

$f_X(x)$ — Raspodela frekvencija jediničnih masa ispitivanog hemijskog elementa (X) u masi probe po nezavisnoj geološkoj promenljivoj (x).

Grafička ilustracija ovih funkcija može biti dvojni poligon, histogram ili dijagram. Na apscisu se nanose vrednosti nezavisne geološke promenljive (x), a na ordinatu vrednosti funkcija $f(x)$ (stubovi) i $f_X(x)$ (puni stubići) kao zavisne geološke promenljive (njihove frekvencije).

Fig. 5. The ideal right-asymmetrical distribution (negative asymmetry) of geological random variable
a) Dual histogram of distribution of functions $f(x)$ (pillars) and $f_X(x)$ (full pillars)
b) Density curves of distribution of functions $f(x)$ and $f_X(x)$
c) Cummulative curves of distribution of functions $F(x)$ and $F_X(x)$

Sl. 5. Idealna desnoasimetrična raspodela (negativna asimetrija) geološke slučajne promenljive
a) Dvojni histogram raspodele funkcija $f(x)$ (stubovi) i $f_X(x)$ (puni stubići)
b) Krive gustine raspodele funkcija $f(x)$ i $f_X(x)$
c) Kumulativne krive raspodele funkcija $F(x)$ i $F_X(x)$

Odnos koncentracija hemijskih elemenata u probi (Na/K, Th/U) takođe ima osobine nezavisne geološke promenljive.

Teorema 3: Statistička razmera je mera standardizacije geološke nezavisne promenljive (x), koja određuje položaj i veličinu svake klase i izolinije koncentracije (geohemijsku ekvidistancu) ispitivanog hemijskog elementa na apscisi histograma (dijagrama) i geohemijskoj karti.

Geohemijska ekvidistanca je interval koncentracije između susednih izolinija. Statistička razmera je odnos jedinice prema geohemijskoj ekvidistanci, a ona može biti linearna (npr. 1 : 1, 1 : 10, 1 : 100 $\mu g/g$) i logaritamska (npr. log 1 : 10, log 1 : 20 $\mu g/g$).

Teorema 4: Raspodela geološke slučajne promenljive je određena sa tri parametra: 1) matematičkim očekivanjem (μ) populacije (aritmetičkom sredinom $\bar{x}$ statističkog uzorka), 2) medijanom (M) i 3) medijanom ispitivanog hemijskog elementa (Mx). Medijane dele geološku populaciju na dve polovine (osiromašenu i obogaćenu) po masi materijala (q) i po masi ispitivanog elementa (X) u probi.

Teorema 5: Matematičko očekivanje (μ) populacije (aritmetička sredina $\bar{x}$ statističkog uzorka) je zajednička tačka (tačka preseka) krivih gustina f(x) i $f_X(x)$ i predstavlja tačku izjednačenja jediničnih relativnih frekvencija $(X_s/\Sigma X = q/\Sigma q = 1/N)$ ovih funkcija.

$$\mu = \frac{\int_a^b f_X(x)\,dx}{\int_a^b f(x)\,dx} \quad ; \quad \bar{x} = \frac{\Sigma X}{\Sigma q} = \frac{\Sigma X}{N} = \frac{\Sigma x}{N}$$

Srednji sadržaj izučavanog hemijskog elementa u geološkom telu E(x) je srednja vrednost geološke slučajne promenljive (matematičko očekivanje μ). Ova veličina je uvek nepoznata, a u praksi se umesto nje uzima aritmetička sredina izučavanog hemijskog elementa u statističkom uzorku (reprezentu geološkog tela).

Teorema 6: Položaj geološke populacije na apscisnoj osi geološke slučajne promenljive (x) je određen intervalom varijacija koncentracija hemijskog elementa (x_{max} — x_{min} kao elementom geohemijskog polja) u probama statističkog uzorka. Van intervala variacija geološka slučajna promenljiva nije definisana.

Tip raspodele geološke slučajne promenljive, odnosno forma krive gustine f(x) je matematički određen položajem parametara raspodele (po teoremi 4) na apscisnoj osi. Zakonima raspodele definisan je tip raspodele geološke slučajne promenljive, i to:

Prvi zakon: Položaj matematičkog očekivanja populacije (aritmetičke sredine statističkog uzorka) na apscisnoj osi kod simetrične raspodele približno se poklapa sa medijanom (M):

$$\bar{x} \approx M < Mx$$

Normalna raspodela je specijalan slučaj simetrične raspodele, kada se matematičko očekivanje (aritmetička sredina) potpuno poklapa (po brojnoj vrednosti) sa medijanom ($\bar{x}$ = M).

Drugi zakon: Položaj matematičkog očekivanja populacije (aritmetičke sredine statističkog uzorka) na apscisnoj osi kod levoasimetrične (pozitivna asimetrija) raspodele je između dve medijane:

$$M < \bar{x} < Mx$$

Primenom logaritamskih odnosa na apscisi (log x) može se eliminisati leva asimetrija krive gustine f(x) i zato se drugi zakon još naziva »logaritamski zakon«.

Treći zakon: Položaj matematičkog očekivanja populacije (aritmetičke sredine statističkog uzorka) na apscisnoj osi kod desnoasimetrične (negativna asimetrija) raspodele je levo od medijana:

$$\bar{x} < M < Mx$$

Za desnoasimetrične raspodele nije pronađen postupak eliminacije asimetrije krive gustine f(x).

Parametri raspodele (po teoremi 4) su jedinstveni za sve tipove krivih gustina, a zakoni raspodele odbacuju mogućnost različitog tretmana pomenutih parametara. Primenom logaritamskih (i eksponencijalnih) odnosa na apscisi nezavisne geološke promenljive (x) služi samo za formalizaciju u smislu postizanja relativne simetrije krive gustine f(x) u odnosu na svoju medijanu (M), a nikako ne daju mogućnost primene geometrijske ili eksponencijalne sredine za izračunavanje srednje vrednosti E(x). Matematičko očekivanje (aritmetička sredina) je jedina vrednost za srednju vrednost E(x) geološke slučajne promenljive, što je definisano teoremom 5.

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